

Development of Students' Ability to Solve Non-Standard Problems in Mathematics As a Means of Encouraging Creative Thinking

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Abstract

This article discusses some non-standard problems and their solution methods in the mathematics course at general secondary schools and academic lyceums with sufficient examples.

Key words: standard problem, non-standard problem, mathematical problem, practical problem, equation, irrational equation, root equation, monotonic function.

It is known that in recent years, attention to education, especially in mathematics, has sharply increased in our country. This can be seen as a concrete example in the resolution No. PQ-4387 of the President of our Republic dated July 9, 2019 [1]. Education in general secondary schools and academic lyceums gains importance in improving mathematical problems. The process of solving problems involves students contemplating, observing, independent exploration, and deep analysis of solutions. Such skills are essential for any professional.

Various pedagogical mathematicians, such as A.D. Semushin, K.I. Neshkov, Yu.M. Kolyagin, J. Ikromov, T. Tolaganov, N.R. Gaybullayev, classify the functions that solve problems and examples in the mathematics course as follows:

1. Educational function of the problem.
2. Upbringing function of the problem.
3. Developmental nature function of the problem.
4. Verifying function of the problem.

Different pedagogical mathematicians give different definitions of the problem concept. For example, S. Aliksonov defines the problem as "The problem is the natural expression of situations occurring in our daily life" [2]. L. M. Fridman and E. N. Turetsky define the problem as "Finding a quantity that satisfies the given and implied conditions" and classify the problems into the following types:

1. With respect to its object (source): practical problems and mathematical problems.
2. With respect to theory: standard and non-standard problems.
3. With respect to requirements: related to calculation, conversion, creation, and proof [3].

The problems mentioned above are utilized in the teaching of mathematics in general secondary schools and academic lyceums. However, it is possible to allocate problems (related to calculation, conversion, creation, and proof) to standard and non-standard problems in the general sense. Therefore, in this article, we firstly explain the content and essence of the concepts of standard and non-standard problems.

If it is possible to prepare and solve such a problem using the laws and rules known in advance and by students, it is called a standard problem [2.30].

The laws and rules emphasized in this definition can include "Word rules," "Formula rules," "Reality rules," "Theorem rules," "Definition rules," "Theorem rules."

If we examine the section on equations as an example,

$$\frac{3+25x}{3x+7} = 5, \quad 2x^2 + 3x - 14 = 0, \quad x^4 - 5x^2 + 6 = 0, \quad (x^2 + 5x + 4)(x^2 + 5x + 6) = 120,$$

$$\frac{x^2 + 1}{x} + \frac{x}{x^2 + 1} = -2,5, \quad |x+4| + |x-2| + |x-3| = 7, \quad x^2 - 3x + \sqrt{x^2 - 3x + 5} = 7, \quad \sqrt{x} + \sqrt[4]{x} = 12,$$

$$\log_x 2 \cdot \log_{2x} 2 = \log_{4x} 2, \quad \sin^2 x + \sin^2 4x = \sin^2 2x + \sin^2 3x$$

and equations are standard problems. If we analyze the process of solving these equations here, we can observe the possibility of establishing general laws and rules and developing a system for solving the problem based on them.

The content and definition of a standard problem is given below in the context of the concept of non-standard problem:

If there are no general rules indicating a specific program for solving a problem in a mathematics course, then such a problem is called a non-standard problem [4.34]. Solving non-standard problems is reduced to solving several standard problems. Here is an example:

Problem 1: At what value of the variable $\frac{y}{y-3}$ and $\frac{6}{y+3}$ the sum of the fractions is equal to their product. Solution:

1) We find the sum of the given fractions: $\frac{y}{y-3} + \frac{6}{y+3} = \frac{y^2 + 3y + 6y - 18}{(y-3)(y+3)} = \frac{y^2 + 9y - 18}{y^2 - 9}$
 2) We find the product of the given fractions:

$$\frac{y}{y-3} \cdot \frac{6}{y+3} = \frac{6y}{y^2 - 9};$$

3) We compare the results of the first and second steps. In this $y^2 - 9 \neq 0$.
 $\frac{y^2 + 9y - 18}{y^2 - 9} = \frac{6y}{y^2 - 9}$, $y^2 + 9y - 18 = 6y$, $y^2 + 3y - 18 = 0$, $y_1 = 3$, $y_2 = -6$.

$y^2 - 9 \neq 0$ only satisfies the condition from the found values of $y = -6$.

Answer: -6.

Analyzing the process of solving the problem, we see that it consists of the following standard problems:

Finding the sum of two fractions (standard problem);

Finding the product of two fractions (standard problem);

Solving the quadratic equation resulting from equating both fractions (standard problem); $y^2 - 9 \neq 0$ condition check (standard issue).

Problem 2: If $\sin 37^\circ = a$, express $\sin 16^\circ$ by a .

Solution: There is no set rule to solve this problem. The student should use the expression given by thinking and observation to express $\sin 16^\circ$ through a . In doing so, he must find a way to generate $\sin 16^\circ$ using $\sin 37^\circ$, that is, notice that the sum of twice 37° and 16° is 90° . This is done as follows:

- 1) $\sin 37^\circ = a$ square both sides of, i.e.; $\sin^2 37^\circ = a^2$;
- 2) Both sides of the resulting equation are multiplied by 2, i.e. $2\sin^2 37^\circ = 2a^2$;
- 3) $2\sin^2 \alpha = 1 - \cos 2\alpha$ The formula is applied to the left side of the resulting equation, i.e. $1 - \cos 74^\circ = 2a^2$ or equality is used $\cos 74^\circ = 1 - 2a^2$;
- 4) $\cos 74^\circ = \sin 16^\circ$ or equality is used. . So, $\sin 16^\circ = 1 - 2a^2$.

Answer: $1 - 2a^2$.

It can be seen that the problem given here is reduced to four standard problems.

Problem 3: Solve the equation $x^2 + \frac{9x^2}{(x+3)^2} = 27$

Solution: If we try to solve this problem as a standard problem, i.e. by bringing to the common denominator and then simplifying, the result will be a quadratic equation and it will be much more difficult to solve. Therefore, it is necessary to do something different here. If we look at the left side

of the given equation, we will see that it is a sum of $(x)^2$ and $\left(\frac{3x}{x+3}\right)^2$ expressions, and we will come to the conclusion that it cannot be squared. To do this, we find that we need to add $-2x \cdot \frac{3x}{x+3}$ to both sides of the equation:

$$x^2 + \frac{9x^2}{(x+3)^2} = 27, \quad x^2 - 2x \cdot \frac{3x}{x+3} + \frac{9x^2}{(x+3)^2} = 27 - 2x \cdot \frac{3x}{x+3}, \quad \left(x - \frac{3x}{x+3}\right)^2 = 27 - 6 \cdot \frac{x^2}{x+3},$$

(here $x \neq -3$).

If we define, $y = \frac{x^2}{x+3}$ then we arrive at Eq. $y^2 + 6y - 27 = 0$. This is a quadratic equation and its roots $y_1 = 3$ and $y_2 = -9$. So we have the following two equations:

$$1) \frac{x^2}{x+3} = 3; \quad 2) \frac{x^2}{x+3} = -9.$$

$$1) \frac{x^2}{x+3} = 3, \quad x^2 - 3x - 9 = 0, \quad x_{1,2} = \frac{3 \pm \sqrt{9+36}}{2} = \frac{3 \pm \sqrt{45}}{2}; \quad x_1 = \frac{3 + \sqrt{45}}{2}, \quad x_2 = \frac{3 - \sqrt{45}}{2}.$$

$$2) \frac{x^2}{x+3} = -9, \quad x^2 + 9x + 27 = 0 \quad . \text{ This equation has no solution.}$$

$$\text{Answer : } \frac{3 \pm \sqrt{45}}{2}.$$

The following theorems are used to solve a number of practical problems:

Theorem 1: If the equation $f(x) = g(x)$ contains $f(x)$ monotonically increasing and $g(x)$ monotonically decreasing, then the solution of the equation $x = x_0$ is unique.

Theorem 2: If the equation $f(x) = g(x)$ is $f(x)$ monotonically increasing and $g(x)$ invariant, then the solution of the equation $x = x_0$ is unique.

Problem 4: solve the equation.. $1 + 3^{\frac{x}{2}} = 2^x$

Solution: The given equation is an exponential equation. It cannot be solved by traditional methods

To solve it, we divide both sides by 2^x and we get the following equation.

$$\left(\frac{1}{2}\right)^x + \left(\frac{1}{\sqrt{3}}\right)^x = 1$$

In this case, we use If we take into account that $\frac{1}{2} = \sin \frac{\pi}{6}$ and $\frac{\sqrt{3}}{2} = \cos \frac{\pi}{6}$ the last equation becomes

$$\left(\sin \frac{\pi}{6}\right)^x + \left(\cos \frac{\pi}{6}\right)^x = 1.$$

Comparing this equation with $\sin^2 x + \cos^2 x = 1$ the real one, we find that it is the root of $x = 2$ the given equation. The given equation has no roots other than $x = 2$. Because the left side of it consists of the sum of two decreasing functions, and the right side consists of a constant number. According to the above theorem, the given equation has a single root $x = 2$.

Answer: 2.

Problem 5: Solve the equation $x^5 + 4x = -40$

. Solution: Let $y_1 = x^5 + 4x$ and $y_2 = -40$.. In this case, the function $y_1 = x^5 + 4x$ is increasing and $y_2 = -40$ is constant. so if the given equation has a root based on the above theorem, it will be unique. We see that $x = -2$ it satisfies the equation given by the selection method. Indeed $(-2)^5 + 4 \cdot (-2) = -32 - 8 = -40, -40 = -40$.

So, $x = -2$ is the root of the given equation

Problem 6: Solve the equation $\sqrt{25 - (2x - 3)^2} = 5 + \sin^2 \frac{2\pi x}{3}$

Solution: The left side of the given equation is an irrational expression, and the right side is a sum of constant numbers with a trigonometric function. Therefore, this equation cannot be called an irrational equation or a trigonometric equation. Such equations are usually called mixed type equations. Such equations are mainly solved by estimation method.

Analyzing the left and right sides of the given equation, we make sure that

$$\sqrt{25 - (2x - 3)^2} \leq 5 \text{ and } 5 + \sin^2 \frac{2\pi x}{3} \geq 5$$

So, the left side of the equation is not greater than 5, and the right side is not less than 5. From this it follows that both sides of the equation are equal to 5, that is, This is done either way. So

$\sqrt{25 - (2x - 3)^2} = 5$. And then $2x - 3 = 0$ or $x = 1,5$. By putting this on the right-hand side of the equation, we make sure that $x = 1,5$ is indeed a root of the equation.

Answer: 1.5.

Problem 7 : Solve the equation $3x^4 - 16x^3 + 18x^2 + 36 = \sqrt{72 + 6x - x^2}$

Solution: The given equation is an irrational equation. If we try to solve this equation by squaring both sides, a more complicated equation of the eighth degree will be formed and it will be difficult to solve. Therefore, traditional methods cannot be used to solve this equation.

Let's look at the function $f(x) = 3x^4 - 16x^3 + 18x^2 + 36$
 $f'(x) = (3x^4 - 16x^3 + 18x^2 + 36)' = 12x^3 - 48x^2 + 36x = 12x(x^2 - 4x + 3) = 12x(x - 1)(x - 3)$. $f'(x) = 0$
 ,, that is, we solve the equation and find the critical points $12x(x - 1)(x - 3) = 0$, $x_1 = 0$, $x_2 = 1$, $x_3 = 3$
 Since $x < 0$ and $f'(x) < 0$; $0 < x < 1$ and $f'(x) > 0$; $1 < x < 3$ and $f'(x) < 0$ and then $3 < x < +\infty$ and $f'(x) > 0$ $f(x)$ the function at $x_1 = 0$ and $x_3 = 3$ has a minimum and at $x_2 = 1$ has a maximum.
 They consist of $f_{\min}(0) = 36$, $f_{\min}(3) = 9$ and $f_{\max}(1) = 41$.. So, the smallest value of the left side of the given equation is 9. Now we find the smallest value of the right side.

$$g(x) = \sqrt{72 + 6x - x^2} = \sqrt{-(x^2 - 6x - 72)} = \sqrt{-(x-3)^2 - 81} = \sqrt{81 - (x-3)^2} \leq 9$$

So, $g_{\min}(x) = 9$. So it is $f_{\min}(x) = g_{\min}(x) = 9$ ekan. Taking these into account, we arrive at the following system:

$$\begin{cases} f(x) = 9 \\ g(x) = 9 \end{cases}$$

Both equations of the system become true equations at $x = 3$

Answer: 3

Problem 8: Solve the equation $x\sqrt{1+x} + \sqrt{3-x} = 2\sqrt{1+x^2}$

. Solution: This equation is an irrational equation. If we square both sides of the given equation in the traditional way, a more complex equation is formed than the given equation and it becomes more difficult to solve it. That is why we use the Cauchy-Buniakovsky inequality (its special case) to solve this equation. It is as follows:

$$a_1 a_2 + b_1 b_2 \leq \sqrt{a_1^2 + b_1^2} \cdot \sqrt{a_2^2 + b_2^2}$$

The geometric meaning of this inequality is as follows: The scalar product of two vectors is not greater than the product of the lengths of these vectors. So we get:

The above inequality becomes an equality when the vectors (a_1, b_1) are (a_2, b_2) are collinear.

So we have

$$x\sqrt{1+x} + 1 \cdot \sqrt{3-x} \leq \sqrt{x^2 + 1} \cdot \sqrt{(1+x) + (3-x)} = 2\sqrt{1+x^2}$$

So $(x, 1)$ and $(\sqrt{1+x}, \sqrt{3-x})$ vectors are collinear

So, $\frac{x}{\sqrt{1+x}} = \frac{1}{\sqrt{3-x}}$, $x\sqrt{3-x} = \sqrt{1+x}$ or $x^3 - 3x + x + 1 = 0$, $(x-1)(x^2 - 2x - 1) = 0$. From this equation, we can write the equations $x-1=0$ and $x^2 - 2x - 1 = 0$, We solve these and find $x_1 = 1$, $x_2 = 1 + \sqrt{2}$, $x_3 = 1 - \sqrt{2}$. $x_3 = 1 - \sqrt{2}$ is The foreign root of these.

Answer: 1 and $1 + \sqrt{2}$.

Problem 9: Solve the equation $8x(2x^2 - 1)(8x^4 - 8x^2 + 1) = 1$

Solution: If $|x| \geq 1$, $2x^2 - 1 \geq 1$ and since $8x^4 - 8x^2 + 1 \geq 1$ the above equality will not hold. So, it should be $|x| < 1$ we define. $x = \cos t$, $0 < t < \pi$. If we consider this definition,

$$2x^2 - 1 = 2\cos^2 t - 1 = \cos 2t, \quad 8x^4 - 8x^2 + 1 = 2(2x^2 - 1)^2 - 1 = 2\cos^2 2t - 1 = \cos 4t$$

$8\cos t \cdot \cos 2t \cdot \cos 4t = 1$ becomes the given equation. We multiply both sides of this equation by $\sin t$

$$8\sin t \cdot \cos t \cdot \cos 2t \cdot \cos 4t = \sin t, \quad 4\sin 2t \cdot \cos 2t \cdot \cos 4t = \sin t, \quad 2\sin 4t \cdot \cos 4t = \sin t,$$

$$\sin 8t = \sin t, \quad \sin 8t - \sin t = 0$$

an equation is formed. To solve this, we multiply its left side. $\sin 8t - \sin t = 0$, $2\cos \frac{9}{2}t \cdot \sin \frac{7}{2}t = 0$.

From this we make the simplest trigonometric equations: $\cos \frac{9}{2}t = 0$ and $\sin \frac{7}{2}t = 0$. We find solutions that satisfy these conditions $0 < t < \pi$.

$$1) \cos \frac{9}{2}t = 0, \frac{9}{2}t = \frac{\pi}{2} + k\pi, 9t = \pi + 2k\pi, t = \frac{\pi}{9} + \frac{2k\pi}{9}, k = 0, k = 1, k = 2, k = 3.$$

$$2) \sin \frac{7}{2}t = 0, \frac{7}{2}t = k\pi, t = \frac{2k\pi}{7}, k = 1, k = 2, k = 3.$$

Answer: $\cos \frac{2\pi}{7}, \cos \frac{4\pi}{7}, \cos \frac{6\pi}{7}, \cos \frac{\pi}{9}, \frac{1}{2}, \cos \frac{5\pi}{9}, \cos \frac{7\pi}{9}.$

Problem 10: Solve the equation. : $x^3 - x - \sqrt{2} = 0$

Solution: Suppose: $x = t\sqrt{2}$. Then the given equation looks like this:

$(t\sqrt{2})^3 - t\sqrt{2} - \sqrt{2} = 0, 2\sqrt{2}t^3 - \sqrt{2}t - \sqrt{2} = 0, 2t^3 - t - 1 = 0.$ To solve the resulting equation, we divide its left side into multipliers.

$$2t^3 - t - 1 = 0, 2t^3 - 2t^2 + 2t^2 - 2t + t - 1 = 0, 2t^2(t - 1) + 2t(t - 1) + (t - 1) = 0, (t - 1)(2t^2 + 2t + 1) = 0.$$

So $t - 1 = 0$ or $2t^2 + 2t + 1 = 0$.. From the first of these we find $t = 1$ The second equation has no solution. So, $x = t\sqrt{2} = \sqrt{2}$

. Problem 11: Solve the equation $\sqrt[3]{2x^2 + 8x + 72} + \sqrt[3]{3x^2 + 12x + 12} = \sqrt{12 - 4x - x^2}$

Solution: This equation is an irrational equation, and since the left-hand side is a sum of cubic roots and the right-hand side is a square root, raising both sides to any degree when solving it results in a more complex equation. That's why we have to do things differently on the ground. First, we change the expressions under the root.

$$\sqrt[3]{2x^2 + 8x + 72} = \sqrt[3]{2(x^2 + 4x + 36)} = \sqrt[3]{2(x + 2)^2 + 64} \geq \sqrt[3]{64} = 4;$$

$$\sqrt[3]{3x^2 + 12x + 12} = \sqrt[3]{3(x^2 + 4x + 4)} = \sqrt[3]{3(x + 2)^2} \geq 0;$$

$$\sqrt{12 - 4x - x^2} = \sqrt{-(x^2 + 4x - 12)} = \sqrt{-(x + 2)^2 - 16} = \sqrt{16 - (x + 2)^2} \leq \sqrt{16} = 4.$$

If we consider the given equation, we can see that for any values on the left side of the given equation, it is not less than 4, and on the right side it is not more than 4. In this case, the equation is satisfied when both sides of the equation are equal to 4, that is

$$\sqrt{12 - 4x - x^2} = 4, 12 - 4x - x^2 = 16, x^2 + 4x + 4 = 0, (x + 2)^2 = 0, x = -2.$$

By checking, we can see the maintenance of the given equation.

Answer: -2.

Problem 12: Solve the equation. $1 + 2^x = \sqrt{33 - x^3}$

Resolution: Let the equation be solved.

$$33 - x^3 \geq 0, x^3 \leq 33, x \leq \sqrt[3]{33}.$$

: When $1+2^x = \sqrt{33-x^3}$, $1+2^x - \sqrt{33-x^3} = 0$. $x \leq \sqrt[3]{33}$ $f(x) = 1+2^x - \sqrt{33-x^3}$ the function is monotonically increasing, Therefore, if the given equation has a root, it is unique By trusting the correctness that the given equation has a root $x=2$.
 $f(2) = 1+2^2 - \sqrt{33-2^3} = 1+4 - \sqrt{33-8} = 5 - \sqrt{25} = 5-5 = 0$.

Answer : 2.

So, we have addressed some non-standard problems that occur in general secondary education and academic lyceums in mathematics courses. Solving these and similar problems requires students to use creativity, innovation, and knowledge of a number of properties of functions. Solving non-standard problems promotes effective learning of all topics in mathematics.

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