

Equation of the Result of Second-Order Surfaces

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Abstract:

In this article, we gave brief information about the product equation of second-order surfaces.

Kali word: hyperbolic paraboloid, ellipsoid, cylindrical surface, function.

A set of points defined by a second-order algebraic equation on an affend is called a second-order surface. The equation of the product plane given in the figure is transferred to the point $M_0(x_0y_0z_0)$

$$(x - x_0)(A_1x_0 + B_1y_0 + B_2z_0 + C_1) + (y - y_0)(A_2y_0 + B_1x_0 + B_3z_0 + C_2) + (z - z_0)(A_3z_0 + B_2x_0 + B_3y_0 + C_3) = 0 \quad (2)$$

we saw that it will look like this. If the left side of (1) is defined as $F(x, y, z)$, and first in (1) x (considering y and z constant), then in y (where x and z are o is considered constant), finally we take derivatives with respect to z (taking x and y as constant).

$$F'_x(x, y, z) = 2 A_1x + 2B_1y + 2B_2z + 2C_1 = 2(A_1x + B_1y + B_2z + C_1),$$

$$F'_y(x, y, z) = 2 A_2y + 2B_1x + 2B_3z + 2C_2 = 2(A_2y + B_1x + B_3z + C_2),$$

$$F'_z(x, y, z) = 2 A_3z + 2B_2x + 2B_3y + 2C_3 = 2(A_3z + B_2x + B_3y + C_3),$$

If we find the values of these functions at the point $M_0(x_0y_0z_0)$

$$F'_{x_0}(x_0, y_0, z_0) = 2(A_1x_0 + B_1y_0 + B_2z_0 + C_1),$$

$$F'_{y_0}(x_0, y_0, z_0) = 2(A_2y_0 + B_1x_0 + B_3z_0 + C_2),$$

$$F'_{z_0}(x_0, y_0, z_0) = 2(A_3z_0 + B_2x_0 + B_3y_0 + C_3),$$

of these

$$\frac{1}{2} F'_{x_0}(x_0, y_0, z_0) = A_1x_0 + B_1y_0 + B_2z_0 + C_1,$$

$$\frac{1}{2}F'_{y_0}(x_0, y_0, z_0) = A_2y_0 + B_1x_0 + B_3z_0 + C_2,$$

$$\frac{1}{2}F'_{z_0}(x_0, y_0, z_0) = A_3z_0 + B_2x_0 + B_3y_0 + C_3,$$

Let's put these values in (2).

$$(x - x_0) \frac{1}{2}F'_{x_0}(x_0, y_0, z_0) + (y - y_0) \frac{1}{2}F'_{y_0}(x_0, y_0, z_0) + (z - z_0) \frac{1}{2}F'_{z_0}(x_0, y_0, z_0)$$

Or to put it more succinctly,

$$F'_{x_0}(x - x_0) + F'_{y_0}(y - y_0) + F'_{z_0}(z - z_0) = 0 \quad (3)$$

This equation is the equation of the test plane transferred to the second-order surface in the most convenient form.

We note that $F'_{x_0}, F'_{y_0}, F'_{z_0}$ cannot be equal to zero at the same time, otherwise the point M will remain a specific point for the surface.

Using equation (3), let's write the equation of the test plane transferred to the second-order surface given by the canonical equations.

a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ Let's write the equation of effort transferred to the point $M_0(x_0y_0z_0)$ of the ellipsoid.

Here

$$F(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0,$$

$$F'_x = \frac{2}{a^2}x, F'_y = \frac{2}{b^2}y, F'_z = \frac{2}{c^2}z.$$

So,

$$F'_{x_0} = \frac{2x_0}{a^2}, F'_{y_0} = \frac{2y_0}{b^2}, F'_{z_0} = \frac{2z_0}{c^2};$$

Putting these in (3) and considering that the point M_0 belongs to the ellipsoid,

$$(x - x_0) \frac{2x_0}{a^2} + (y - y_0) \frac{2y_0}{b^2} + (z - z_0) \frac{2z_0}{c^2} = 0,$$

From this

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} + \frac{zz_0}{c^2} = 1. \quad (4)$$

b) $\frac{x^2}{p} - \frac{y^2}{q} = 2z$ let's write the equation of the test plane at the point $M_0(x_0y_0z_0)$ of the hyperbolic paraboloid. Here

$$F(x, y, z) = \frac{x^2}{p} - \frac{y^2}{q} - 2z = 0,$$

$F'_x = \frac{2}{p}x, F'_y = \frac{2}{q}y, F'_z = -2$, their values at point M_0

$$F'_{x_0} = \frac{2}{p}x_0, F'_{y_0} = \frac{2}{q}y_0, F'_{z_0} = -2$$

Let's put these in (3).

$$\frac{2}{p}x_0(x - x_0) + (-\frac{2}{q}y_0)(y - y_0) + (-2)(z - z_0) = 0$$

If we consider that M_0 belongs to this surface, the sought equation will be as follows:

$$\frac{xx_0}{p} - \frac{yy_0}{q} = z + z_0. \quad (5)$$

c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ let's write the equation of the test plane on the cylindrical surface at its point $M_0(x_0y_0z_0)$.

$$F'_x = \frac{2}{a^2}x, F'_y = \frac{2}{b^2}y, F'_z = 0;$$

their values at the point $M_0(x_0y_0z_0)$ are $F'_x = \frac{2}{a^2}x, F'_y = \frac{2}{b^2}y, F'_z = 0$, put it in (3) if

$$\frac{2}{a^2}x_0(x - x_0) + \frac{2}{b^2}y_0(y - y_0) + 0(z - z_0) = 0$$

Or

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} = 1. \quad (6)$$

s) $\frac{x^2}{p} + \frac{y^2}{q} = 2z$ Let's write the equation of the plane of the elliptic paraboloid at the point $M_0(x_0y_0z_0)$. Here

$$F(x, y, z) = \frac{x^2}{p} + \frac{y^2}{q} - 2z = 0$$

$F'_x = \frac{2}{a^2}x, F'_y = \frac{2}{b^2}y, F'_z = -2$; if we put these in (3) and take into account that the point M_0 belongs to the elliptic

$$\frac{2}{p}x_0(x - x_0) + \frac{2}{q}y_0(y - y_0) + (-2)(z - z_0) = 0$$

From this

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} = 3z - z_0. \quad (7)$$

Besides these, we can prove any second-order surfaces by formula (3).

Example 1 $\frac{x^2}{27} + \frac{y^2}{12} + \frac{z^2}{75} = 1$ construct the equation of the product plane at the point $M(3,2,5)$ of the ellipsoid

Solution: if we transfer this example to the formula (4) derived from (3).

$$\frac{3x}{27} + \frac{2y}{12} + \frac{5z}{75} = 1 \leftrightarrow \frac{x}{9} + \frac{y}{6} + \frac{z}{15} = 1 \leftrightarrow 10x + 15y + 6z - 90 = 0 \text{ the plane equation is derived.}$$

References

1. N.D.Dadajonov. M.SH.Jo'raeva "Geometriya" 1-qism Toshkent-"O'qituvchi" 1982-y
2. S.V.Baxvalov. P.S. Modenov. A.S.Parxomenko Analitik geometriyadan misol va masalalar to'plami. Rus tilidan tarjima Toshkent, Universitet 2005-y
3. Qori Niyoziy T.N Analitik geometriya asosiy kursi fan 1971-y
4. T.H.Rasulov. G'.G'.Qurbonov.Z.N.Hamdamiy. Analitik geometriyadan misol va masalalar to'plami

5. B.M.Rustamov. J.U.To`raxonov. Kasodlik ehtimolligining aniq hisobiga doir misol. Образование наука и инновационные идеи в мире 35(2) 172-175
6. B.M.Rustamov. Sug`urta kompaniyasi kasodligining dinamik modellari Значение цифровизации и искусственного интеллекта в устойчивом развитии. Международная совместная научно-практическая конференция
7. B.M.Rustamov. J.U.To`raxonov. Kasodlik ehtimolligining aniq hisobiga doir misol. Образование наука и инновационные идеи в мире 35(2) 172-175
8. B.M.Rustamov. Sug`urta kompaniyasi kasodligining dinamik modellari Значение цифровизации и искусственного интеллекта в устойчивом развитии. Международная совместная научно-практическая конференция 15.03.2024-у 273-bet
9. Abdurashidov N.G'., Simmetrik Li va Leybnits algebralari va ularning xossalari. "O'ZBEKISTONDA FANLARARO INNOVATSIYALAR VA ILMIY TADQIQOTLAR" APREL 2022. (7), 62-63.
10. Eshtemirov Eshtemir Salim o'g'li, Abdurashidov Nuriddin G'iyoziddin o'g'li. VEYL-TITCHMARSH FUNKSIYASI VA SPEKTRAL FUNKSIYA ORASIDAGI MUNOSABAT. "O'ZBEKISTONDA FANLARARO INNOVATSIYALAR VA ILMIY TADQIQOTLAR" 20-iyun 2023-yil 20-son (870-875).
11. Abdurashidov N. G'., Eshtemirov E. S .SIMMETRIK LEYBNITS ALGEBRALARI VA ULARNING XOSSALARI. << Matematik modellashirish va axborot texnologiyalarining dolzarb masalalari >> xalqaro ilmiy-amaliy anjuman. Nukus 2-3-may 2023-yil 1-Tom.
12. Abdurashidov Nuriddin , Toshtemirova Sarvara, Yo'ldoshev Husan.
13. Laplas teoremasi yordamida 4-tartibli determinantni hisoblash. "O'ZBEKISTONDA FANLARARO INNOVATSIYALAR VA ILMIY TADQIQOTLAR" 20-fevral 2024-yil 27-son (163-166).
14. Abdurashidov Nuriddin, O'ktamova Nigora, Ahmadova Sevinch. "Geometriya masalalarini yechishda vektorlarning ba'zi bir tadbiqlari". Pedagog respublika ilmiy jurnali. 15-mart 2024-yil 7-tom 3-son (61-66).
15. Rustamov Bilol, O'ktamova Nigoraxon, Ahmadova Sevinch. ikkinchi tartibli sirtlarning urunma tenglamasi."So`ngi ilmiy tadqiqotlar nazaryasi"
16. Mirsoburova D.M, Togayev T.X, Toshtemirov U.E. Об одной композиции несобственных интегралов со слабыми особенностями. 2022-йил Термиз.
17. Тогаев Т.Х. О потенциале двойного слоя для уравнения Геллерстедта с сингулярным коэффициентом. СОВРЕМЕННЫЕ ПРОБЛЕМЫ ТЕОРИИ ЧИСЕЛ И МАТЕМАТИЧЕСКОГО АНАЛИЗА. ДУШАНБЕ 2022