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Complex Numbers and their Characteristics

Ibodullayeva Farangiz Sheraliyevna [1], Kurbanov Gulomjon Gafurovich [2]

[1] Bukhara State Pedagogical Institute, 4th level student

[2] Bukhara State Pedagogical Institute, Associate Professor of the Department of Exact Sciences, p.f.f.d (PhD

Abstract:

This article delves into the world of complex numbers. We define them, explore their algebraic representation, and study the fundamental operations that govern their interactions. Additionally, we introduce the geometric interpretation of complex numbers, envisioning them as points in the complex plane, and gaining insight into their magnitude and direction.

Keywords: Complex number, imaginary unit, real and imaginary parts of a complex number, complex conjugate number, complex plane, real and imaginary axes, absolute value and argument of a complex number, trigonometric form of a complex number, theorem on the absolute value of a sum.

Introduction.

The field of mathematics extends beyond the familiar world of real numbers into a fascinating and powerful concept: complex numbers. Composed of real and imaginary parts, these numbers open new avenues in mathematical research and find applications across various fields, from physics and engineering to signal processing and computer science. This article explores the fundamental aspects of complex numbers, providing comprehensive information about their definitions, operations, and key properties [1].

We start by defining the system of complex numbers as an extension of the system of real numbers, introducing the concept of the imaginary unit "i". Next, we investigate the algebraic representation of complex numbers, focusing on addition, subtraction, multiplication, and division. We also explore the geometric interpretation of complex numbers using the complex plane, allowing us to visualize their magnitude and argument. Furthermore, we introduce important concepts related to the modulus of complex numbers and highlight their properties and significance. Finally, we study the trigonometric form of complex numbers and provide a brief overview of the powerful Moivre's formula for calculating powers and roots of complex numbers [2].

This article serves as a fundamental resource for understanding the basics of complex numbers, equipping learners with the essential knowledge to explore their applications and appreciate their rich mathematical structure [3].

Main Part.

Complex Numbers in Algebraic Form and Operations on Them.

Let R be the set of real numbers. We define addition and multiplication operations on the set $C = R \times R$ as follows:

$(a, b) + (c, d) = (a + c, b + d)$,

$(a, b) \cdot (c, d) = (ac - bd, bc - ad)$.

Clearly, the following conditions hold for the addition and multiplication defined in C:

a) Commutativity of Addition: $(a, b) + (c, d) = (c, d) + (a, b)$

b) Associativity of Addition:

$[(a, b) + (c, d)] + (e, f) = (a, b) + [(c, d) + (e, f)]$,

c) Commutativity of Multiplication: $(a, b) \cdot (c, d) = (c, d) \cdot (a, b)$

d) Associativity of Multiplication:

$[(a, b) \cdot (c, d)] \cdot (e, f) = (a, b) \cdot [(c, d) \cdot (e, f)]$.

The validity of these laws follows from the corresponding equations.

e) Distributive Law:

$[(a, b) + (c, d)] \cdot (e, f) = (a, b) \cdot (e, f) + (c, d) \cdot (e, f)$;

It is also easy to verify the validity of this distributive law:

$$[(a, b) \cdot (c, d)] \cdot (e, f) = (a + c, b + d) \cdot (e, f) =$$

$= (ae + ce - bf - df, af + cf + be + de)$,

$$(a, b) \cdot (e, f) + (c, d) \cdot (e, f) = (ae - bf, af + be) + (ce - df, cf + df) =$$

$= (ae + ce - bf - df, af + cf + be + de)$.

It is noteworthy that $(0,0)$ is the trivial (zero) element of the set C, and $(1,0)$ is the multiplicative identity, i.e. [4]:

$(a, b) + (0,0) = (0,0) + (a, b) = (a, b)$,

$(a, b) \cdot (1,0) = (1,0) \cdot (a, b) = (a, b)$.

It is known that any element $(a, b) \in C$ has an opposite element $(-a, -b)$. Now we will show that any non-zero element $(a, b) \in C$ has an inverse, i.e., the equation $(a, b) \cdot (x, y) = (1,0)$ has a solution. From this equation, we have:

$$(ax - by, bx + ay) = (1,0).$$

This gives rise to the following system of two equations:

$$\begin{cases} ax - by = 1 \\ bx + ay = 0. \end{cases}$$

It is known that this system has a solution when $(a, b) \neq (0,0)$, and that the solutions are given by:

$x = \frac{a}{a^2+b^2}, y = \frac{-b}{a^2+b^2}$.

Thus, for the element (a, b), the inverse element is given by

$(a, b)^{-1} = \left(\frac{a}{a^2+b^2}, -\frac{b}{a^2+b^2}\right)$.

Definition 1: The set of complex numbers is defined as the set C with addition, subtraction, multiplication, and division operations. The elements of the set of complex numbers are called complex numbers [5].

We denote the set of elements of the form $(a, 0)$ in the set of complex numbers as R_1 Let's observe the addition and multiplication operations defined in R_1:

$$(a, 0) + (c, 0) = (a + c, 0)$$

$(a, 0) \cdot (c, 0) = (ac, 0)$.

From these equations, we can see that the addition and multiplication operations in the set R_1 are defined in the same way as in the set of real numbers. If we establish a mapping $f: R_1 \to R$ such that $f\big((a, 0)\big) = a$, it follows from the above equations that this mapping preserves the operations of addition and multiplication. Therefore, we can assume that $(a, 0) = a$.

If we denote the element $(0,1)$ by i, then we have:

$i^2 = (0,1) \cdot (0,1) = (-1,0) = -1$.

This element $i \in C$ is called the imaginary unit. For any $(a, b) \in C$, we can write:

$(a, b) = (a, 0) + (0, b) = (a, 0) + (b, 0) \cdot (0,1) = a + bi$.

Thus, any element of the set of complex numbers C can be expressed in the form $z = a + bi$. This form is called the algebraic form of a complex number. The number in the algebraic form of a complex number is called the real part of the complex number and is denoted as $Re(z)$. The number b is called the imaginary part of the complex number z and is denoted as $Im(z)$. Complex numbers for which the imaginary part is equal to zero are real numbers, while complex numbers for which the real part is zero are called imaginary complex numbers. The complex conjugate of a complex number $z = a + bi$ is denoted as $\bar{z} = a - bi$. For complex conjugates, the following equalities hold:

$$z + \bar{z} = (a + bi) + (a - bi) = 2a,$$

$z \cdot \bar{z} = (a + bi) \cdot (a - bi) = a^2 + b^2$,

meaning that the sum and product of a complex number and its conjugate are real numbers.

Property 1: The sum of complex numbers has the following properties:

a) $\overline{z_1 + z_2} = \bar{z_1} + \bar{z_2}$;

b) $\overline{z_1 - z_2} = \bar{z_1} - \bar{z_2}$;

c) $\overline{z_1 \cdot z_2} = \bar{z_1} \cdot \bar{z_2}$;

d) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z_1}}{\bar{z_2}}$

Using the conjugate is very convenient for finding the inverse of a complex number[6]:

$$(a + bi)^{-1} = \frac{1}{a + bi} = \frac{1}{a + bi} \cdot \frac{a - bi}{a - bi} = \frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2} i.$$

Review Questions:

- Describe the extension of the field of real numbers.
- How are complex numbers formed?

- Define the arithmetic operations on complex numbers.
- Prove that the set of complex numbers forms a field.
- Prove the properties of the sum of complex numbers.
- Define the modulus of a complex number.
- Prove the properties of the modulus of a complex number.

In today's world, knowing a foreign language is becoming a global demand, which makes it important to examine the terminology related to this subject in foreign languages, particularly in English, and to pay special attention to their explanations to reinforce your knowledge [7].

Integration implies the interconnection of multiple directions or branches in any field.

Terms	**Explanation**
Complex number	An expression in the form $a + bi$ (where aaa and b are real numbers, and i is the imaginary unit) is called a complex number. The number a is the real part of the complex number, while bi is its imaginary part.
Two equal complex numbers	Two complex numbers $a + bi$ and $s + di$ are equal if and only if $a = s$ and $b = d$.
Joint complex numbers	Complex numbers of the form $a + ib$ and $a - ib$ are called joint complex numbers
The formule of Muavr	$z^n = (r(\cos\varphi + i\sin\varphi))^n = r^n(\cos n\varphi + i\sin n\varphi)$

Conclusion

This article provided information about complex numbers, their properties, and arithmetic operations. Basic arithmetic operations such as addition, subtraction, multiplication, and division on complex numbers were clearly demonstrated based on specific rules. The importance of learning the terminology related to complex numbers in English was also emphasized. This process aids in the expansion of mathematical knowledge. Overall, complex numbers play a crucial role in mathematics and other sciences, being applied not only in theoretical problems but also in practical situations [1-10]. This article encompasses fundamental concepts that help in understanding complex numbers and their properties.

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