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Methods for Solving Odd-Degree Inverse Equations, Denau Institute of Entrepreneurship and Pedagogy

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Abstract:

This article provides an overview of odd-degree recurrent equations, focusing specifically on their fundamental characteristics, mathematical properties, and potential applications. Recurrent equations, especially those of odd degree, have unique symmetric properties and behavior patterns that distinguish them from even-degree counterparts. Despite their importance, odd-degree recurrence equations often receive limited attention in existing literature, necessitating a clearer understanding of their distinctive features. In this paper, we discuss general forms of odd-degree recurrent equations, examine their symmetry and structural properties, and highlight key distinctions from recurrent equations of even degree. Additionally, illustrative examples are presented to clarify theoretical concepts and to demonstrate practical applications of these equations. Through this examination, the article aims to facilitate deeper insights into odd-degree recurrence equations, thereby encouraging further research and application in relevant areas of mathematical and computational science.

Keywords: Odd-degree equations, recurrent equations, symmetry, higher-order equations, mathematical analysis

Introduction.

The study of recurrent equations plays an essential role in various branches of mathematics, physics, engineering, and applied sciences. In particular, recurrent equations, which are equations defined in terms of their preceding values, frequently appear in modeling complex systems, dynamical processes, and computational algorithms. Among these, odd-degree recurrent equations form a special class due to their unique symmetry and mathematical properties. Despite their practical relevance and theoretical significance, odd-degree recurrent equations remain less explored compared to their even-degree counterparts, presenting opportunities for further scholarly investigation. Odd-degree recurrent equations inherently possess symmetrical features, enabling specific techniques and strategies for their analytical and numerical solutions. These equations inherently exhibit roots with symmetrical properties, typically characterized by pairs of positive and negative solutions. Recognizing and leveraging this symmetry can simplify the problem-solving process and facilitate a deeper understanding of underlying structures.This paper aims to address existing gaps in the literature by thoroughly examining methods for solving odd-degree recurrent equations. Emphasis is placed on exploring the symmetry, root structures, and algebraic transformations of these equations. Additionally, various solution techniques are illustrated through practical examples to clarify the underlying concepts and demonstrate their applicability. By providing a comprehensive analysis and accessible approach, this article seeks to enhance both theoretical understanding and practical capabilities in addressing odd-degree recurrence equations, ultimately encouraging broader utilization and study in mathematical and applied contexts.

$a_0 x^{2n+1} + a_1 x^{2n} + a_2 x^{2n-1} + \ldots + a_n x^{n+1} + l\, a_n x^n + l^3 a_{n-1} x^{n-1} + a_0 l^{2n+1} = 0$ (1)

$a_0 x^{2n} + a_1 x^{2n-1} + a_2 x^{2n-2} + \ldots + a_{n+1} x^{n+1} + a_n x^n + l\, a_{n-1} x^{n-1} + l^2 a_{n-2} x^{n-2} + \ldots + l^n a_0 = 0$ (2) [1,2]

In this λ- a fixed number and $a_0 \neq 0$, equations don't go back equations is called . λEquations (1) and (2) at =1 odd and pair level symmetrical equations It is called . Odd level don't go back equation every always $x = -\lambda$ to the root has , because this equation following in appearance to write possible .

$a_0 (x^{2n+1} + \lambda^{2n+1}) + a_1 x (x^{2n-1} + \lambda^{2n-1}) + \ldots + a x^n (x + \lambda) = 0.$

Bracket at x = λ− inside every one expression to 0 becomes . x +λ multiplier from the bracket outside subtract , equation (1) x + λ= 0 and pair level don't go back equations to the union equal strong x = 0 (2) of the equation root absence for him/her x^n to we will be and terms in groups

$a_0 (x^n + (\frac{\lambda}{x})^n) + a_1 (x^{n-1} + (\frac{\lambda}{x})^{n-1}) + \ldots + a_{n-1} (x + \frac{\lambda}{x}) + a_n = 0$

equation harvest we do . $x+\frac{\lambda}{x}$ = to mark as we enter .

$x^2 + (\frac{\lambda}{x})^2 = u^2 - 2\lambda.$

$x^3 + (\frac{\lambda}{x})^3 = (x+ \frac{\lambda}{x}) - 3\lambda (x+ \frac{\lambda}{x}) = u^3 - 3u\lambda,$

$x^4 + (\frac{\lambda}{x})^4 = (x+ \frac{\lambda}{x})^4 - 4\lambda (x^2 + (\frac{\lambda}{x})^2) - 6\lambda^2 = u^4 - 4u^2\lambda + 2\lambda^2$

and etc. In this case , for x of the 2nth degree relative equation (3) to n-th degree u related algebraic equation in appearance writing we get . Harvest which is n - degree equation solution possible If , equation (2) roots are also found .

$ax^3 + bx^2 + bx + a = 0, a \neq 0$ (4)

in appearance equations third level don't go back symmetrical equations is called . [3-6]

$ax^3 + bx^2 + bx + a = a(x^3 + 1) + bx(x + 1) = a(x+1)(x^2 - x + 1) + bx(x+1) =$

$= (x+1)(ax^2 + (b - a)x + a)$

that was for equation (4)

$x + 1 = 0$ and $ax^2 + (b - a)x + a = 0$

of the equation to the union equal strong will be . [7 -14]

Example . This

$x^5 + 3x^4 - x^3 + 2x^2 - 24x - 32 = 0$

equation solve it .

Solution: This equation fifth level don't go back equation is , $\lambda = -2$ equals . The equation as follows writing we get :

$x^5 + 3x^4 - x^3 - x^2 \cdot(-2) + 3x\cdot(-2)^3 + (-2)^5 = 0.$;

The equation $x = 2$ root it will be , from now on left side of the equation terms we group :

$x^5 - 32 + 3x^4 - 24x - x^3 + 2x^2 = 0$

$(x^5 - 32) + 3x(x^3 - 8) - x^2(x - 2) = 0$

To the multipliers we separate :

$(x - 2)(x^4 + 2x^3 + 4x^2 + 8x + 16) + 3x(x - 2)(x^2 + 2x + 4) - x^2(x - 2) = 0$

$(x - 2)(x^4 + 5x^3 + 9x^2 + 20x + 16 = 0$

1. $x - 2 = 0 \Rightarrow x_1 = 2.$
2. The equation $x^4 + 5x^3 + 9x^2 + 20x + 16 = 0$ fourth level don't go back equation is , in this $\lambda = 4$ equal

$x^4 + 5x^3 + 9x^2 + 5x4 + 4^2 = 0$

The equation $x = 0$ root not . The equation both side x^2 to we will be :

$x^2 + 5x + 9 + \frac{20}{x} + (\frac{4}{x})^2 = 0$ or $(x^2 + (\frac{4}{x})^2) + 5(x + \frac{4}{x}) + 9 = 0.$

Define $x + \frac{4}{x} = y$ we enter and $x^2 + (\frac{4}{x})^2 = y^2 - 8$ harvest we will do .

Found last to the equation we pour :

$y^2 - 8 + 5y + 9 = 0 \Rightarrow y^2 + 5y + 1 = 0.$

The following to the equation We are coming. This equation roots we find : $D = 25 - 4 = 21 > 0$.

$y_{1.2} = \frac{-5 \pm \sqrt{21}}{2} \Rightarrow y_1 = \frac{-5 - \sqrt{21}}{2}; y_2 = \frac{-5 + \sqrt{21}}{2}.$

So , the equation following equations to the couple equal strong :

1. $x + \frac{4}{x} = \frac{-5-\sqrt{21}}{2} \Rightarrow 2x^2 + (5+\sqrt{21})x + 8 = 0.$
2. $x + \frac{4}{x} = \frac{-5+\sqrt{21}}{2} \Rightarrow 2x^2 + (5-\sqrt{21})x + 8 = 0.$

Second equation real to the roots has not . The first one roots

$x1 = \frac{-5-\sqrt{21}-\sqrt{10\sqrt{21}-18}}{4}$; $x2 = \frac{-5-\sqrt{21}+\sqrt{10\sqrt{21}-18}}{4}$

Answer : $\left\{2, \frac{-5-\sqrt{21}-\sqrt{10\sqrt{21}-18}}{4}, \frac{-5-\sqrt{21}+\sqrt{10\sqrt{21}-18}}{4}\right\}.$

Conclusion.

The study of odd-degree recurrent equations provides essential insights into their unique characteristics, solution methods, and practical implications. This paper has highlighted the fundamental differences between odd and even-degree equations, emphasizing the intrinsic symmetry and consistent presence of negative roots in odd-degree equations. Through illustrative examples, including specific methods for grouping terms and applying substitutions, we demonstrated practical approaches to solving these equations. The analysis underscores the importance of recognizing structural symmetry and distinct mathematical behaviors that arise in odd-degree equations compared to even-degree ones. Understanding these peculiarities can significantly simplify solving processes and open avenues for further research. Future studies should explore additional computational methods and their implications across applied mathematics and computational sciences. Enhanced comprehension of odd-degree recurrence equations promises valuable contributions to fields requiring advanced mathematical modeling, thereby enriching theoretical knowledge and practical applicability in various scientific and engineering contexts.

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