EXCELLENCIA
INTERNATIONAL MULTI-DISCIPLINARY JOURNAL OF EDUCATION
Volume 02, Issue 03, 2024 ISSN (E): 2994-9521

Solving Differential Equations Using the Operational Method In the Maple System

Mirzakarimov Ergashboy Mirzaboevich
Fergana Polytechnic Institute Assistant Professor
Fergana Polytechnic Institute, Fergana, Uzbekistan
-mail: ergashboy.mir@mail.ru

Fayzullayev Djamshid Ismoiljonovich
Acting Associate Professor
Fergana Polytechnic Institute, Fergana, Uzbekistan
fayzullayev4748@gmail.com , j.fayzullayev@ferpi.uz
ORCID:0000-0002-3753-6736
Tel: 936437980

Abstract

There are different approaches to solving DE. With a known solution algorithm, it can be implemented “manually,” which is usually quite labor-intensive. You can develop a computer implementation of the algorithm in any programming language. This approach involves labor-intensive writing and debugging of programs. The third approach is to use existing computer mathematics systems, which have built-in procedures for implementing the necessary algorithms. This approach currently seems to be the most rational from the point of view of minimizing time costs and avoiding errors.

Key words: operator, original, image, inttrans linearity, initial, boundary, convert, invlaplace, pdsolve, dsolve, simplify, subs.

Introduction. Solving differential equations (DE). The Maple system has built-in numerous algorithms for solving DE. With its help, you can solve single and systems of differential equations of various orders, both ordinary and partial derivatives.

Statement of the problem. One of the most important applications of operational calculus associated with the Laplace transform is the solution of linear differential equations with constant coefficients.

When practically solving differential equations using the operational method, it is, of course, recommended to use ready-made correspondence tables between originals and images. The more detailed these tables are, the less additional calculations and transformations you will have to do.

Problem solution.1. Let's a second-order linear differential equation with constant coefficients is given

1. Let's a second-order linear differential equation with constant coefficients is given

$$x''(t)+a_1x'(t)+a_2x(t)=f(t) \qquad (1)$$

and initial conditions: $x(0)=x_0,\ x'(0)=x'_0$. We will assume that the required function $x(t)$, its derivatives $x'(t), x''(t)$ and $f(t)$ function is original and $x(t)\leftarrow X(p)$ and $f(t)\leftarrow F(p)$. Then, by the differentiation property of the original $x'(t)\leftarrow pX(p)-x_0$, $x''(t)\leftarrow p^2X(p)-px_0-x'_0$.

Next, using the linearity property of the original, instead of differential equation (1) with initial conditions, we obtain the operator equation

$$p^2X(p)-px_0-x'_0+a(pX(p)-x_0)+a_2X(p)=F(p)$$

$$(p^2+a_1p+a_2)X(p)=F(p)+px_0+x'_0+a_1x_0$$

Let's resolve it relatively $X(p)$:

$$X(p)=\frac{F(p)+px_0+x'_0+a_1x_0}{p^2+a_1p+a_2}$$

If we find the original from the image, then this will be the desired particular solution. Find the solution to the equation $\ddot{x}+4\dot{x}+4x=t^3e^{-2t}$ at initial conditions $x(0)=1,\ \dot{x}(0)=2.$

Let's first find the image of the right side. Because $t^3\div\frac{3!}{p^4}$, then by the damping theorem $t^3e^{-2t}\div\frac{3!}{(p+2)^4}$. Considering that $\dot{x}(t)\div pX(p)-1$ and $\ddot{x}(t)\div p^2X(p)-p-2$, Let's create an operator equation:

$$p^2X(p)-p-2+4pX(p)-4+4X(p)=\frac{3!}{(p+2)^4};$$

from here $$X(p)=\frac{3!}{(p+2)^6}+\frac{p+6}{(p+2)^2}.$$

The original for the first term is found using the formula given above. Because $\frac{5!}{p^6}\div t^5$, то $\frac{3!}{(p+2)^6}\div\frac{3!}{5!}t^5e^{-2t}$ (image displacement theorem).

We write the second term in the form $\frac{p+2+4}{(p+2)^2}=\frac{1}{p+2}+\frac{4}{(p+2)^2}$.

Then $\frac{1}{p+2} \div e^{-2t}$, $\frac{4}{(p+2)^2} \div 4te^{-2t}$, and finally

$$x(t)=e^{-2t}+4te^{-2t}+\frac{1}{20}t^5e^{-2t}=e^{-2t}\left(1+4t+\frac{t^5}{20}\right).$$

2. Define the operator solution to differential equation (1) and an example in the **Maple system:**

```
> restart;
> ur:=diff(x(t),t$2)+a[1]*diff(x(t),t)+ a[2]*x(t)=f(t);
```

$$ur := \frac{d^2}{dt^2}x(t)+a_1\left(\frac{d}{dt}x(t)\right)+a_2x(t)=f(t)$$

```
> #diff(x(t),t)=p*X(p)-x[0];
> d1:=p*X-x0;
```

$$d1 := pX-x0$$

```
> #diff(x(t),t,t)=p^2*X(p)-p*x[0]-x1[0];
> d2:=p^2*X-p*x0-x10;
```

$$d2 := p^2X-p\,x0-x10$$

```
> x1:=X;   f1:=F(p);
```

$$x1 := X f1 := F(p)$$

```
> d2+a[1]*d1+a[2]*x1=f1;
```

$$(Xp-x0)\,a_1+Xa_2+Xp^2-p\,x0-x10=F(p)$$

```
> solve(d2+a[1]*d1+a[2]*x1=f1,{X});
```

$$\left\{X=\frac{p\,x0+x0\,a_1+F(p)+x10}{p^2+p\,a_1+a_2}\right\}$$

```
> X:=(p*x0+x0*a[1]+F(p)+x10)/(p^2+p*a[1]+a[2]);
```

$$X := \frac{p\,x0+x0\,a_1+F(p)+x10}{p^2+p\,a_1+a_2}$$

Example solution:

```
> a[1]:=4:a[2]:=4: x0:=1:  x10:=2: f(t):=t^3*exp(-2*t);
```

$$f(t) := t^3\,e^{-2t}$$

```
> ur:=diff(x(t),t$2)+a[1]*diff(x(t),t)+a[2]*x(t)=f(t);
```

$$ur := \frac{d^2}{dt^2}x(t)+4\left(\frac{d}{dt}x(t)\right)+4x(t)=t^3\,e^{-2t}$$

```
> with(inttrans): F(p):=laplace(f(t),t,p);
```

$$F(p) := \frac{6}{(p+2)^4}$$

```
> X:=(p*x0+x0*a[1]+F(p)+x10)/(p^2+p*a[1]+a[2]);
```

$$X := \frac{p + 6 + \frac{6}{(p+2)^4}}{p^2 + 4p + 4}$$

> factor(X);

$$\frac{p^5 + 14p^4 + 72p^3 + 176p^2 + 208p + 102}{(p+2)^6}$$

> with(inttrans);

$[addtable, fourier, fouriercos, fouriersin, hankel, hilbert, invfourier, invhilbert, invlaplace, invmellin, laplace, mellin, savetable]$

> F(p):=convert(X,parfrac,p);

$$F(p) := \frac{4}{(p+2)^2} + \frac{6}{(p+2)^6} + \frac{1}{p+2}$$

> x(t):=invlaplace(F(p),p,t);

$$x(t) := \frac{1}{20} e^{-2t} (20 + 80t + t^5)$$

You can see that if you solve this equation using conventional methods, it will require much longer calculations.

Analysis of the results obtained. The Malpe system can be used to quickly, accurately and efficiently solve problems in the differential equation sections of mathematical analysis of higher mathematics. Malpe makes it possible to solve engineering problems and create animated graphics and figures in 2D and 3D[1,3,4,5,7,8,9].

Conclusion. The Malpe system has recently become useful for many users personal computers that engage in mathematical calculations, ranging from solving educational problems in universities to modeling complex physical objects, systems and devices. Undoubtedly, any scientific laboratory or university department should have a mathematical system if they are seriously interested in automating the performance of mathematical calculations of any degree of complexity[4].

We offer a training manual created by A.K. Urinov and M.E. Mirzakarimov and published by the Ministry of Higher Education, is a textbook “Ordinary Differential Equations in the Maple System”, which is useful for teachers when teaching “Higher Mathematics” using Maple [15].

Literatures

1. Матросов А.В. Maple 6. Решение задач высшей математики и механики. СПб.: БХВ - Петербупг, 2001.
2. Мирзакаримов Е.М. Решение задачи по высшей математике с помощью Maple. 1,2,3q, Т.: "Искры литературы", 2014,2015,2015.
3. Мирзакаримов Е.М. Решение задач аналитической геометрии в системе Maple. Часть 1, Т.: "Навруз", 2019 г.
4. Mirzakarimov, E. M., & Fayzullaev, J. S. (2020). Improving the quality and efficiency of teaching by developing students* mathematical competence using the animation method of

adding vectors to the plane using the maple system. *Scientific Bulletin of Namangan State University*, *2*(9), 336-342.

5. Mirzakarimov E. M., Faizullaev J. I.,(2019) Method of teaching the integration of information and educational technologies in a heterogeneous parabolic equation //scientific bulletin of namangan state university. т. 1. – №. 5. – с. 13-17.
6. Fayzullaev, J. (2020). A systematic approach to the development of mathematical competence among students of technical universities. *European Journal of Research and Reflection in Educational Sciences Vol*, *8*(3). 42-47
7. Fayzullayev J. I. Mathematical competence development method for students through solving the vibration problem with a Maple system //scientific bulletin of namangan state university. – 2020. – т. 2. – №. 8. – с. 353-358.
8. Mirzaboevich, M. E., & ugli, E. M. I. (2021). Using Maple Programs in Higher Mathematics. Triangle Problem Constructed on Vectors in Space. *Central Asian Journal Of Mathematical Theory And Computer Sciences*, *2*(11), 44-50. Retrieved from https://cajmtcs.centralasianstudies.org/index.php/CAJMTCS/article/view/123
9. Mirzakarimov EM. (2022).To ‘g ‘riburchakli membrananing erkin tebranishini boshlang ‘ich shartlar bo ‘yicha aniqlashda maple tizimidan foydalanish. Scientific Bulletin of Namangan State University. 1(1):53-61.
10. .Мирзабоевич, М. Э. (2022). Использовать Систему Maple Для Определения Свободных Колебаний Прямоугольной Мембраны При Начальных Условиях. *Central Asian Journal Of Mathematical Theory And Computer Sciences*, *3*(1), 9-18. Retrieved from https://cajmtcs.centralasianstudies.org/index.php/CAJMTCS/article/view/148
11. Mirzaboyevich, M. E., & ugli, E. M. I. (2022). Using the Maple System to Evaluate the Efficiency of a Regression Model. *Central Asian Journal Of Mathematical Theory And Computer Sciences*, *3*(5), 7-13. https://doi.org/10.17605/OSF.IO/Q4JRC
12. Mirzaboyevich, M. E. (2022). Using the Maple System in Selecting an Efficient Model for the Analysis of Experimental Results. *Central Asian Journal Of Mathematical Theory And Computer Sciences*, *3*(5), 14-27. https://doi.org/10.17605/OSF.IO/J8VZM.
13. Mirzakarimov Ergashboy Mirzaboyevich, F. D. I. (2023). Maple Tizimida Kompleks Sonlarlarning Geometrik Tasviri. *Eurasian Journal Of Mathematical Theory And Computer Sciences*, 3(5), 38–45. https://doi.org/10.5281/zenodo.7923713
14. Mirzakarimov Ergashboy Mirzaboyevich, F. D. I. (2023). Algebraik Shakildagi Kompleks Sonlar Ustida Amallarni Maple Tizimida Bajarish. *Eurasian Journal Of Mathematical Theory And Computer Sciences*, 3(5), 30–37. https://doi.org/10.5281/zenodo.7923704
15. Уринов А.К., Мирзакаримов Е.М. Обыкновенные дифференциальные уравнения в системе Maple. F.: "Fergana", 2020 г.