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The Gronwall Escape Problem for Bounded Second-Order Motions

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Abstract:

In the theory of differential games, the issues of geometric, integral and their joint limitations have been sufficiently studied. In this lecture, the escape problem of the second-order differential game is studied, with the introduction of new control classes under the name of Granwall-type boundedness to the control functions. The theory of differential games is considered today as an important link in the theory of mathematical management as a theory widely studied at the international level and applied in various fields. This dissertation is also devoted to the study of topical issues of differential games, which explores chase-escape issues for players acting with acceleration, i.e. chase-escape Masas of second-order differential games. In this case, various delimitations are considered to the controls. It is expected to implement parallel chase srategia in the solution of the chase issue. It is planned to find the necessary and sufficient conditions for eviction issues.

Keywords: Differential game, acceleration, Granwall's lemma, Granwall's limit, escaper, chaser, initial state, parallel pursuit strategy.

Introduction

Let $\mathbf{P}$ and $\mathbf{E}$ objects with opposite aim be given in $\boldsymbol{R}^n$ space and their movements based on the following differential equations and initial conditions

$$\mathbf{P}: \ddot{x} = u,\ x_1 - kx_0 = 0,\ |u(t)|^2 \le \rho^2 + 2l\int_0^t |u(s)|^2\, ds, \qquad (1)$$

$$\mathbf{E}:\ \ddot{y}=v,\ y_1-ky_0=0,\ |v(t)|^2\le\sigma^2+2l\int_0^t|v(s)|^2\,ds, \qquad (2)$$

where $x,y,u,v\in \boldsymbol{R}^n$; x – a position of **P** object in $\boldsymbol{R}^n$ space, $x_0=x(0),\ x_1=\dot{x}(0)$ – its initial position and velocity respectively at $t=0$; u – a controlled acceleration of the pursuer, mapping $u:[0,\infty)\to\boldsymbol{R}^n$ and it is chosen as a measurable function with respect to time; we denote a set of all measurable functions $u(\cdot)$ such that satisfies the condition $|u(t)|^2\le\rho^2+2l\int_0^t|u(s)|^2\,ds$ by G_P. y – a position of **E** object in $\boldsymbol{R}^n$ space, $y_0=y(0),\ y_1=\dot{y}(0)$ – its initial position and velocity respectively at $t=0$; v – a controlled acceleration of the evader, mapping $v:[0,\infty)\to\boldsymbol{R}^n$ and it is chosen as a measurable function with respect to time; we denote a set of all measurable functions $v(\cdot)$ such that satisfies the condition $|v(t)|^2\le\sigma^2+2l\int_0^t|v(s)|^2\,ds$ by G_E.

Research Methods and the Received Results

Definition 1. For a trio of $(x_0,x_1,u(\cdot)),u(\cdot)\in G_P$, the solution of the equation (1), that is, $x(t)=x_0+x_1t+\int_0^t\int_0^s u(\tau)d\tau ds$ is called a trajectory of the pursuer on interval $t\ge 0$.

Definition 2. For a trio of $(y_0,y_1,v(\cdot)),v(\cdot)\in G_E$, the solution of the equation (2), that is, $y(t)=y_0+y_1t+\int_0^t\int_0^s v(\tau)d\tau ds$ is called a trajectory of the evader on interval $t\ge 0$.

Definition 3. The pursuit problem for the differential game (1) - (2) is called to be solved if there exists such control function $u^*(\cdot)\in \boldsymbol{G_p}$ of the pursuer for any control function $v(\cdot)\in\boldsymbol{G_E}$ of the evader and the following equality is carried out at some finite time t^*

$$x(t^*)=y(t^*). \qquad (3)$$

Definition 4. For the problem (1)-(2), time T is called a guaranteed pursuit time if it is equal to an upper boundary of all the finite values of pursuit time t^* which the equality (3) is true.

Definition 5. For the differential game (1) - (2), the following function is called Π-strategy of the pursuer ([3]-[4]):

$$u(v) = v - \lambda(v)\xi_0, \tag{4}$$

where $\xi_0 = \dfrac{z_0}{|z_0|}$, $\lambda(v) = (v,\xi_0) + \sqrt{(v,\xi_0)^2 + \delta e^{2lt}}$, $\delta = \rho^2 - \sigma^2 \geq 0$,

(v,ξ_0) is a scalar multiplication of vectors v and ξ_0 in the space $\boldsymbol{R^n}$.

Lemma 1 (Granwoll). Suppose, let a mapping $\varphi(t):[0,\infty) \to \boldsymbol{R^n}$ be bounded, nonnegative and measurable function. Moreover, $l \geq 0$ and $\rho > 0$ are constant and for the given if an inequality $|\varphi(t)|^2 \leq \rho^2 + 2l\int_0^t |\varphi(s)|^2 ds$ is carried out, then a relation $\varphi(t) \leq \rho e^{lt}$ is always true.

Lemma 2. If $\rho \geq \sigma$, then the following inequality is true for the function $\lambda(v,\xi_0)$:

$$e^{lt}(\rho-\sigma) \leq \lambda(v,\xi_0) \leq e^{lt}(\rho+\sigma).$$

Theorem. If for the second order differential game (1) – (2) with Granwoll constraint a condition $\rho > \sigma$ is true, then the pursuit problem is solved by Π-strategy (4) on interval $(0,t)$ and an approach function between the objects becomes as follows:

$$f(l,t,|z_0|,\rho,\sigma,k) = |z_0|(kt+1) - \frac{\rho-\sigma}{l^2}e^{lt} + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l}t$$

Conclusion

Proof. Suppose, let the pursuer choose a strategy in the form (4) when the evader chooses any control function $v(\cdot) \in \boldsymbol{G_E}$. Then according to the equations (1) and (2) we define the following Caratheodory's equation

$$\ddot{z} = -\lambda(v(t))\xi_0, \qquad \dot{z}(0) - kz(0) = 0,$$

Hence the following solution will be found by the given initial conditions

$$z(t) = z_0(kt+1) - \xi_0 \int_0^t\int_0^s \lambda(v(\tau),\xi_0)\, d\tau ds \qquad \text{or}$$

$$\left|z(t)\right|=\left|z_0\right|(kt+1)-\int_0^t\int_0^s((v,\xi_0)+\sqrt{(v,\xi_0)^2+\delta e^{2lt}})d\tau ds.$$

We form the following inequalities in relation to **Lemma 1**

$$\left|z(t)\right|\le\left|z_0\right|(kt+1)-\int_0^t\int_0^s e^{l\tau}(\rho-\sigma)d\tau ds\Rightarrow$$

$$\left|z(t)\right|\le\left|z_0\right|(kt+1)-\frac{\rho-\sigma}{l^2}e^{lt}+\frac{\rho-\sigma}{l^2}+\frac{\rho-\sigma}{l}t$$

If we say $f(l,t,\left|z_0\right|,\rho,\sigma,k)=\left|z_0\right|(kt+1)-\dfrac{\rho-\sigma}{l^2}e^{lt}+\dfrac{\rho-\sigma}{l^2}+\dfrac{\rho-\sigma}{l}t$ (5),

then define a positive solution t^* such that the function (5) equals to zero

$$\frac{\rho-\sigma}{l^2}e^{lt}=\left|z_0\right|(kt+1)+\frac{\rho-\sigma}{l^2}+\frac{\rho-\sigma}{l}t.$$

We will form the following equality by simplifying the latest relation

$$e^{lt}=t\left(\frac{\left|z_0\right|kl^2}{\rho-\sigma}+l\right)+\frac{\left|z_0\right|l^2}{\rho-\sigma}+1$$

where $A=\dfrac{\left|z_0\right|kl^2}{\rho-\sigma}+l$, $B=\dfrac{\left|z_0\right|l^2}{\rho-\sigma}+1$, $B>1$. Therefore, we have the following equation

$$e^{lt}=At+B \qquad (6)$$

In order to define a pursuit time we will consider some cases of the equation (5).

1. Let be $A<0\Rightarrow k<\dfrac{\sigma-\rho}{\left|z_0\right|l}$. Then the equation (5) has a unique positive solution t^* and this solution is a pursuit time (Fig-1).

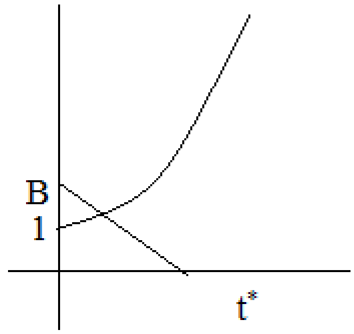

Figure-1

2. Let be $A=0 \Rightarrow k=\dfrac{\sigma-\rho}{|z_0|l}$. Then a solution of the equation (5) is

$t^* = \dfrac{\ln\left(\dfrac{|z_0|l^2}{\rho-\sigma}+1\right)}{l}$, and this solution is a pursuit time (Fig-2).

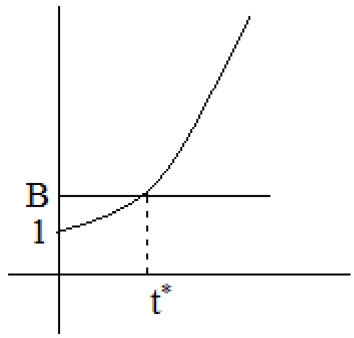

Figure-2

3. Let be $A>0 \Rightarrow k>\dfrac{\sigma-\rho}{|z_0|l}$. Then the equation (5) has a positive solution t^* and this solution is a pursuit time (Fig-3).

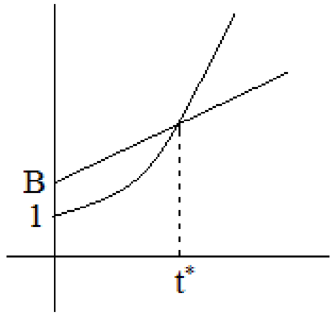

Figure-3

In conclusion, the relation (3) is true in all values of interval $t \geq 0$ according to the inequality $|z(t)| \leq f(k,t,\rho,\sigma,l,|z_0|)$ and properties of (5), i.e., the evasion problem is solved. Proved.

Summary

In the theory of differential games, issues of chase and escape occupy a special place in Aloxi. One of them is the breadth of implementation of various methods, as well as the specificity of the results obtained. This feature is especially obvious in model questions. In accordance with the condition given in the lemma, the theorem is conditioned and provides a proof. In the theory of differential games, questions in which geometric, integral and their joint constraints are imposed on controls have been sufficiently studied. This article includes new control classes in a control function called delimitation, using Gromwell's lemma. The chase-escape problem in a second-order differential game was studied, and new adequacy conditions were proposed for the pursuer and the evader.

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