

Integration of the Spline Function and Fuzzy Logic Algorithm in the Management of Clean Water Filling in a Smart Home System

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Abstract:

This research paper presents a mathematical model for controlling the level of clean water in the capacity of a smart home system, integrating a PID controller, cubic splines and fuzzy logic algorithms. The water filling control model in the tank contains an equation of water level dynamics, which takes into account the rate of water intake and outflow, and the use of a PID controller adjusts the water supply rate. Cubic splines provide an approximation of the water level, and fuzzy logic algorithms optimize control. The result of the study confirms the high accuracy and adaptability of the proposed approach, which ensures optimal resource management in smart home systems [1,2,3].

Keywords: water level control, mathematical modeling, splines, PID controllers, fuzzy logic, adaptive filters.

Introduction

Managing and controlling the filling of clean water in a tank is considered a very important task, especially when it comes to a smart home system. This is necessary to stabilize the water supply

process and optimize the use of water resources. This article discusses the process of controlling water filling using the integration of spline functions as a mathematical model and fuzzy logic algorithms.

Setting the task.

It is necessary to develop a control system [4, 5] for filling a tank with clean water, which takes into account the following parameters:

- Tank volume - V
- Current water level - $h(t)$
- Filling rate - $Q_{in}(t)$
- Outflow rate - $Q_{out}(t)$

Our task is to maintain the water level in the tank at a given level h_{set} with control of changes in the outflow rate and all other factors.

A mathematical model. The basic equation of the dynamics of the water level in the tank [7.10]:

$$\frac{dh(t)}{dt} = \frac{Q_{in}(t) - Q_{out}(t)}{A} \quad (1)$$

Here A is the cross-sectional area of the tank.

Filling equation:

$$Q_{in}(t) = K_p(h_{set} - h(t)) + K_i \int_0^t (h_{set} - h(\tau)) d\tau + K_d \frac{d}{dt} (h_{set} - h(t)) \quad (2)$$

In turn, here K_p, K_i, K_d are the coefficients of the PID controller.

The use of splines. To solve the problem of integrating equations and approximating water level curves, we will use cubic splines.

Cubic splines. The cubic spline $S_i(t)$ on the interval $[t_i, t_{i+1}]$ is given by the following expression [8, 12, 13]:

$$S_i(t) = a_i + b_i(t - t_i) + c_i(t - t_i)^2 + d_i(t - t_i)^3 \quad (3)$$

Let's have this $\{(t_0, h(t_0)), (t_1, h(t_1)), \dots, (t_n, h(t_n))\}$ a set of points. The system of equations for the spline coefficients will look like this:

$$\begin{cases} S_i(t) = h(t_i) \\ S_i(t_i) = h(t_{i+1}) \\ S_i'(t_{i+1}) = S_{i+1}'(t_{i+1}) \\ S_i''(t_{i+1}) = S_{i+1}''(t_{i+1}) \end{cases} \quad (4)$$

Decision:

Consider a specific example where the filling time is $t_0 = 0, t_1 = 1, t_2 = 2$ and the corresponding water levels are $h(0) = 10, h(1) = 15, h(2) = 12$

Interval $[0, 1]$:

$$S_0(t) = a_0 + b_0(t - 0) + c_0(t - 0)^2 + d_0(t - 0)^3$$
$$\begin{cases} S_0(0) = 10 \Rightarrow a_0 = 10 \\ S_0(1) = 15 \Rightarrow 10 + b_0 + c_0 + d_0 = 15 \\ S'_0(1) = S'_1(1) \\ S''_0(1) = S''_1(1) \end{cases}$$

Let's solve the equation: $10 + b_0 + c_0 + d_0 = 15 \Rightarrow b_0 + c_0 + d_0 = 5$

Interval $[1, 2]$:

$$S_1(t) = a_1 + b_1(t - 1) + c_1(t - 1)^2 + d_1(t - 1)^3$$
$$\begin{cases} S_1(1) = 15 \Rightarrow a_1 = 15 \\ S_1(2) = 12 \Rightarrow 15 + b_1 + c_1 + d_1 = 12 \\ S'_1(1) = S'_1(1) \\ S''_1(1) = S''_1(1) \end{cases}$$

Let's solve the equation: $15 + b_1 + c_1 + d_1 = 12 \Rightarrow b_1 + c_1 + d_1 = -3$

Smoothness conditions:

$$b_0 + 2c_0 + 3d_0 = b_1$$

$$2c_0 + 6d_0 = 2c_1$$

Fuzzy Logic algorithms

Fuzzy logic can also be used to control the water level. Let's consider a simple system with two fuzzy rules [9, 5]:

1. If the water level is low, then increase the filling rate.
2. If the water level is high, then reduce the filling rate.

Fuzzification

For the variables $h(t)$ and $Q_{in}(t)$, we define fuzzy sets:

- Low level - $\mu_{low}(h)$

- High level - $\mu_{high}(h)$

Membership functions can be set, for example, by triangular functions.

An example of membership functions.

for a low level:

$$\mu_{low}(h) = \begin{cases} 1 & h \leq 10 \\ \frac{15-h}{5} & 10 < h < 15 \\ 0 & h \geq 15 \end{cases}$$

for a high level:

$$\mu_{high}(h) = \begin{cases} 0 & h \leq 10 \\ \frac{h-10}{5} & 10 < h < 15 \\ 1 & h \geq 15 \end{cases}$$

Application of the rules:

1. If $\mu_{low}(h)$ is maximal, then $Q_{in}(t)$ increases to the maximum value of $Q_{max}(t)$.
2. If $\mu_{high}(h)$ is maximal, then $Q_{in}(t)$ decreases to the minimum value of $Q_{min}(t)$.

Defuzzification. Let's apply the center of gravity method to obtain the final value of $Q_{in}(t)$:

$$Q_{in}(t) = \frac{\int_{Q_{min}}^{Q_{max}} q \cdot \mu(q) dq}{\int_{Q_{min}}^{Q_{max}} \mu(q) dq} \quad (5)$$

When defuzzification by the center of gravity method, it is important to correctly take into account the accessory functions for different water levels. Before proceeding, you need to make sure that all membership functions and integrals are correctly defined and calculated. Let's take a closer look at the whole process.

Using the example of membership functions for the low level $\mu_{low}(h)$ and for the high level $\mu_{high}(h)$, we calculate the membership function.

Defuzzification by the center of gravity method

The membership function $\mu(q)$ for the filling rate $Q_{in}(t)$ can be represented as:

$$\mu(q) = \max(\mu_{low}(h), \mu_{high}(h)) \quad (6)$$

Consider the value of the water level $h = 13$:

$$\text{for a low level: } \mu_{low}(12) = \frac{15-13}{5} = 0,4$$

$$\text{For a high level: } \mu_{high}(12) = \frac{13-10}{5} = 0,6$$

Thus, the membership function $\mu(q)$ for $Q_{in}(t)$:

$$\mu(q) = 0,6 \quad \text{for } q \text{ in the range from 0 to 100}$$

After that, we calculate the center of gravity method using the following formula (5):

Substituting the values:

$$Q_{in}(t) = \frac{0,6 \cdot \int_0^{100} q \, dq}{0,6 \cdot \int_0^{100} dq}$$

Shortening 0.6:

$$Q_{in}(t) = \frac{\int_0^{100} q \, dq}{\int_0^{100} dq}$$

Calculate the integrals:

$$\int_0^{100} q \, dq = \left[\frac{q^2}{2} \right]_0^{100} = \frac{100^2}{2} = 5000$$

$$\int_0^{100} dq = [q]_0^{100} = 100$$

Substituting the values into the formula:

$$Q_{in}(t) = \frac{0,6 \cdot 5000}{0,6 \cdot 100} = 50$$

The application of the center of gravity method for defuzzification in fuzzy logic allows us to obtain the final value of the filling rate $Q_{in}(t) = 50$ at the water level $h=13$. Thus, it can be said with confidence that the use of splines and the fuzzy logic algorithm are very effective in clean water filling control systems in smart home systems [11,14,15], which provides more accurate and adaptive process control.

Conclusion. In this article, we have reviewed the methods of controlling the filling of clean water into a container in a smart home system. To do this, PID regulators were chosen as a mathematical model and cubic splines were used to approximate the dependence of the water level on time, and fuzzy logic was used to calculate the water filling rate based on the current water level. This integrated method makes it possible to create a more accurate and stable management system, which allows for increased savings in the use of water resources and stable operation of the system.

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