Innovative: International Multi-disciplinary Journal of Applied Technology
(ISSN 2995-486X) VOLUME 03 ISSUE 12, 2025

Fractal Analysis Using the Mass-Radius Method

[1]Sh.A.Anarova, [2]D.N. Boliyeva
[1]Tashkent University of Information Technologies named after Muhammad al-Khwarizmi, anorova@tuit.uz
[2]Scientific Research Institute for the Development of Digital Technologies and Artificial Intelligence, boliyevadilrabo38@gmail.com

Abstract:

Biological images often contain structural formations, especially those derived from neural tissues. It is known that neurons are divided into several types, but distinguishing these types remains a constant problem. The main feature of the fractal dimension is proposed in this article. The possibilities of performing fractal analysis using the mass-radius method are investigated below. The obtained results were analyzed based on the mass-radius method. In addition, the article presents the capabilities and characteristics of the software tool designed for calculating the mass-radius method.

Keywords: fractal analysis, complex structures, mass-radius method.

Introduction

The Mass-Radius method is a technique for calculating the fractal dimension of a fractal structure by measuring how the mass (the number of pixels occupied by the structure) changes as the radius increases from the geometric center.

This method is applied in medical imaging, particularly for analyzing:

- The vascular system,
- The bronchial tree of the lungs,
- The neural network of the brain — all of which represent branched and complex forms.

Naturally occurring objects, along with cells and tissues examined in pathology, possess highly intricate structural features that are challenging to represent with Euclidean geometry. Biological systems frequently display restricted self-similar growth, which can be more accurately described by the fractal dimension (F) through fractal geometry. A unique characteristic of fractal geometry is that the assessment of a shape or object relies on the scale at which it is evaluated compared to the object's length [1]. This reliance on measurement is indicated by the object's fractal dimension. The kind of fractal dimension indicates the selfsimilarity of an object or the constant nature of the dimension type, and its most basic representation is expressed as follows:

$$F = \frac{\log(Ns)}{\log(Fm)} \quad (1)$$

In this context, Ns represents the count of self-similar sections, while Fm denotes the magnification coefficient. The fractal dimension F can be determined by utilizing the number of paths, the mass-radius approach, and cube calculations [2].

ANALYSIS OF THE MASS-RADIUS METHOD.

The mass-radius method is carried out using the simplest algorithm and following the lead of "Caserta" + "Eldred." Here, the ratio between the box-counting area in the pixel at one position of the image and the radius, which encloses the image defines the fractal dimension. M grows with the radius intuited:

$$\text{“}M = \mu r^F\text{”}(2)[6]$$

In most cases that we know of, the mass dimension (of self-similar mathematical fractals) is the same as the "Hausdorff dimension" [3]. The circles are placed on points (most likely established in 1 part of the rotation radius) and at every addition all the circle with one pixel. It produces a series of larger nested circles, as shown in Figure 1. In all stages, the sum of the number of pixels of any circle is counted. It stops when it has reached the edge of the image, and stops when it no longer intersects with a pixel of the image. The total value is then calculated for the radius that corresponds to each pixel. To extract the gradient fractal dimension: The slope of the log-log relationship of cumulative mass versus cumulative radius is determined through linear regression (Figure 1).

Figure 1. The placement of centers used to calculate the fractal dimension using the mass-radius method.

For the purpose of fractal dimension evaluation in the next subsection, binary images shown on the computer monitor (obtained from a digital camera, image scanner or high-resolution digitizer) are assumed to represent two-dimensional objects. The mass-radius M(r) relation is M(r)∝r^2 for two-dimensional Euclidean shapes. As disks increase in size, the area of a square is related to the radius of the measurement disk squared. Therefore, the exponent 2 corresponds to the Euclidean dimension (a square is two-dimensional), whereas the mass of a fractal object varies with a fractional exponent (1<F<21 < F < 21<F<2) as follows:

$$M(r) \propto r^F \quad (3)$$

and the fractal dimension is taken as FFF:

$$F = \frac{\log(M(r))}{\log(r)} \quad (4)$$

Graphing r - log, the slope of the line (M(r), input log(r). Minimum error calculation Similarly C(r), the two-point correlation function with respect to M(r), can also be employed to make a prediction regarding the fractal dimension. In the case of a fractal, C(r) decays as a powerlaw, a is a measure of distance:

$$C(r) \propto r^{a} \quad (5)$$

Here, a (the "Euclidean dimension" - fractal dimension) and the two-point correlation function are evaluated as follows:

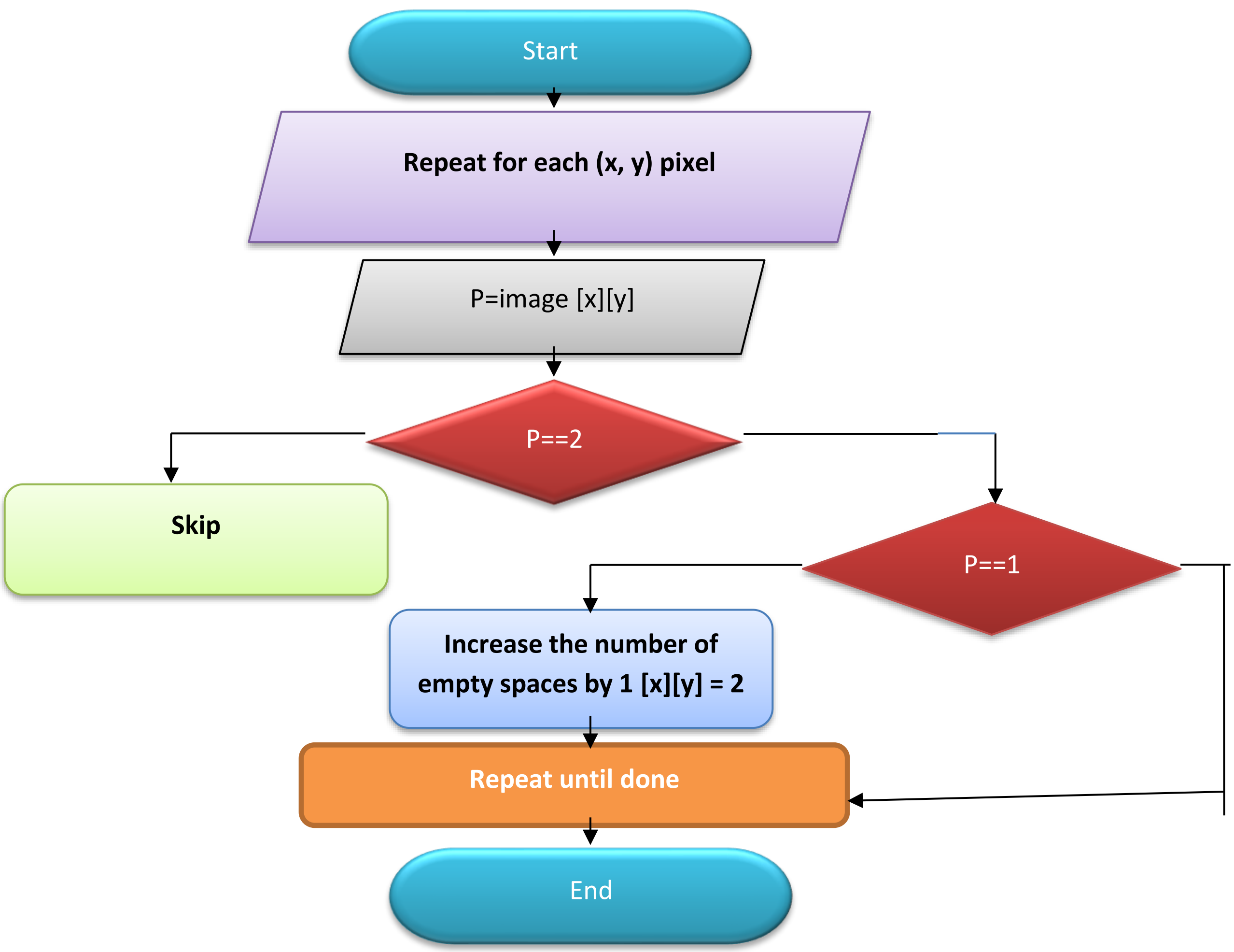
$$C(r) = N^{-1} \sum_{r'} |p(r + r')p(r')|) \; (6)$$

N number of pixels, and the p function the probability of finding a site connected between r and r + r′ bookkeeping (hits minus the distance from the starting point of the measurement: for p, 1 connected sites, 0 unconnected sites unterlass click to expand) If it is a two-dimensional image, then Euclidean dimension is 2. Therefore, we use regression analysis log(C(r)) with respect to log(r) to obtain a letter (F-2) representing fractal dimension as follows:

$$F = 2 + (\frac{\log(C(r))}{\log(r)}) \quad (7)$$

The mass-radius is usually used as a single approximation for the initial position (often at the mass or center of symmetry), C(r) at average initial points across different objects.

To prevent counting pixels multiple times, the algorithm:



Note: pixel = 0 represents an empty site, and pixel = 1 represents a filled site.

Figure 2. The points represent the skipped coordinates during "Bresenham circle" scanning. For a maximum scan radius of 200, the total number of skipped pixels is 12,552.

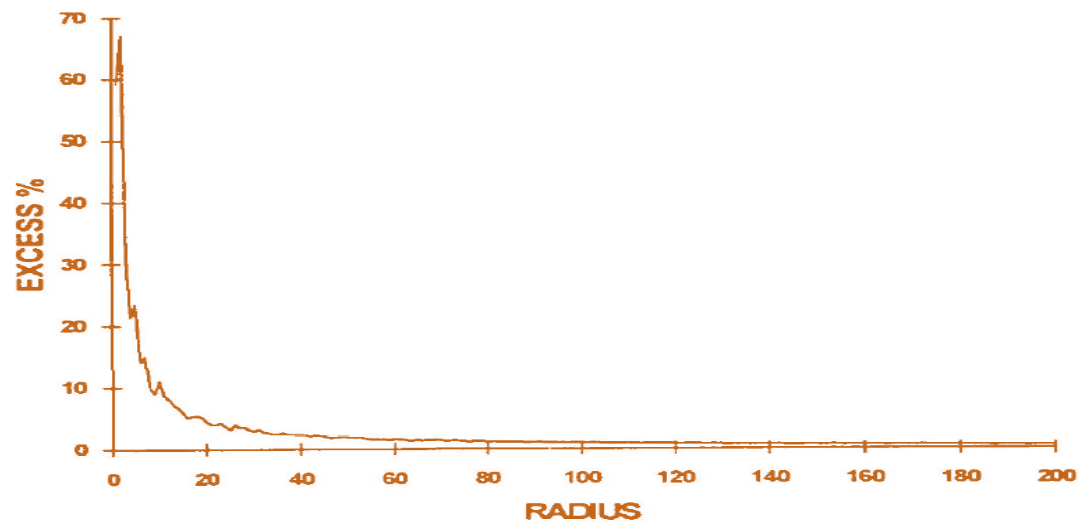


Figure 3. Overestimation of mass, expressed as a percentage of the theoretical circle value.

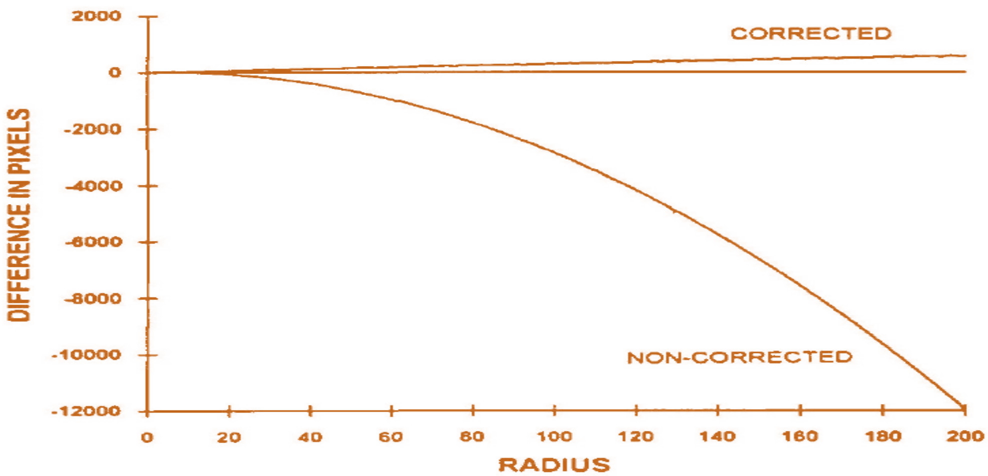


Figure 4. Difference in the number of pixels between the theoretical mass-radius relationship.

The Pythagorean method for drawing circles is sufficient for a computer because it makes extensive use of multiplication functions and square roots. In addition, the slope of the drawing is not linear, which results in large gaps during scanning.

Table 1. Results of fractal dimension determined using the mass-radius method.

Interval r	F (slope)	R^2	y (intercept)
Theoretical circle	2	100	0.497149
1…10	1.805142	99.8286	0.720844
10…20	1.923770	99.9828	0.613958
20…30	1.970078	99.9884	0.554663
30…40	1.976108	99.9932	0.545138
40…50	1.978808	99.9945	0.540946
50…60	1.979512	99.9957	0.539948
60…70	1.986823	99.9934	0.527174
70…80	1.987101	99.9967	0.526518
80…90	1.990119	99.9975	0.520782
90…100	1.989151	99.9955	0.522738
100…110	1.992703	99.9974	0.515622
110…120	1.990857	99.9985	0.519374
120…130	1.993757	99.9982	0.513397
130…140	1.994773	99.9975	0.511178
140…150	1.994502	99.9971	0.511739
150…160	1.993930	99.9985	0.512975
160…170	1.994041	99.9978	0.512710
170…180	1.993729	99.9984	0.513478
180…190	1.994451	99.9978	0.511843
190…200	1.993815	99.9980	0.513309

R^2= Coefficient of determination %.

$$y = r\sin(\theta) \quad (8)$$
$$y = r\cos(\theta) \quad (9)$$

$\theta = o\ \ \pi / 2$ Circle symmetry completed up to.

CONCLUSION.

In this study, we examined the reading of medical images using the Fractop program and the determination of fractal dimensions of planes at various radius ranges using the mass-radius method. When identifying the geometric center of the image, either the image center or the center of the largest component was used. For different radii, M(r) was calculated, the radii were incremented (for example: 10, 20, 30...), and the number of black pixels within the circle for each radius was determined and analyzed.

‘

List of References

1. Mandelbrot, (1983) “The Fractal Geometry of Nature.” W.H. Freeman and Co., New York.
2. researchoutput.csu.edu.au
3. Schroeder, M. (1991). “Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise.” W.H. Freeman & Co.

4. Jelinek, H.F. and Fernandez, E. (2001) "Use of fractal theory in neuroscience: methods, advantages and potential problems." Methods, 24(4):309-321.
5. Einstein AJ. Wu HS. Sanchez M. and Gil J. (1998) "Fractal characterisation of chromatin appearance for diagnosis in breast cytology." Journal of Pathology. 185(4):366-381.
6. John W. Rippin. "Notes on the implementation of the mass – radius method of fractal dimension estimation."