Innovative: International Multi-disciplinary Journal of Applied Technology
(ISSN 2995-486X) VOLUME 03 ISSUE 4, 2025

MATHEMATICAL ECONOMICS AND ECONOMIC RELATIONS IN TERMS OF EQUATIONS

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Abstract:
This study explores the critical role of mathematical modeling in economic theory by analyzing the structure, classification, and application of economic equations in mathematical economics. Despite the growing integration of quantitative approaches in economics, there remains a knowledge gap regarding the systematic use of behavioral, definitional, and technical equations to model complex economic phenomena. This research employs a qualitative, theory-driven methodology, synthesizing classical and modern economic models such as consumption functions and the Cobb-Douglas production function. The findings reveal that mathematical models—ranging from linear and nonlinear to logarithmic and exponential forms—enable the abstraction and simplification of economic relationships, making complex dynamics more analyzable. Results emphasize that the use of mathematical equations enhances the precision and predictive power of economic theories. The implications suggest a deeper alignment between mathematical and econometric methods in policy evaluation, economic forecasting, and structural analysis, reinforcing the foundational role of mathematics in contemporary economic inquiry.

Keywords: Mathematical Economics, Economic Modeling, Behavioral Equations, Econometric Analysis, Cobb-Douglas Function, Linear Regression, Functional Relationships, Economic Forecasting, Technical Equations, Quantitative Methods

1. Introduction

The mathematical method is the best way to provide a comprehensive and accurate picture of economic theory, as this theory provides some information that is useful in determining the mathematical form. However, experimenting with different mathematical formulas and choosing the one that yields the most reasonable results remains the best method. Furthermore, it is the contemporary method for understanding, analyzing, and formulating theoretical aspects, particularly complex ones.

The need to apply mathematical methods in economics arose in order to understand the nature of economic relations. Therefore, these methods emerged as quantitative means of persuading verbal economic relations. Furthermore, mathematical methods have been useful in separating economics from politics, separating the term "political economy" from economics and transforming it into quantitative economics, or what is known as "mathematical economics." In light of the above, what distinguishes the use and application of mathematical methods in economics is the use of mathematical language through what we call mathematical modeling of economic relationships. This involves using a mathematical model to explain and describe the behavior of economic systems and phenomena, whether large or small. These systems express an economic variable, phenomenon, or intertwined relationships across all sectors of the national economy. These models aim to clarify or demonstrate the validity of a theory.

Economic Models

An economic model, in its simplest definition, is a set of relationships that connect a group of economic variables, expressed in the form of equations that explain the relationship between these variables. It can also be a "simplified representation of a real phenomenon."

An economic model is generally considered the primary tool in the field of mathematical economics, consisting of mathematical equations that simplify economic reality. The purpose of its development is to facilitate the description of the nature of the relationship or relationships between economic variables and to measure their mutual impact, in a manner devoid of detail and complexity, yet representing reality as best as possible. Furthermore, the relationship between variables is not limited to just two variables; rather, there may be a group of variables that influence a particular variable. Thus, the mathematical formula expresses the relationship between two or more variables. This depends on the nature of the relationship between the variables and the extent to which it can be formulated mathematically using equations and symbols. By collecting the mathematical formulas, what is called a mathematical model is formed. Therefore, mathematical models are models that aim to clarify or prove the validity of what a basic theory goes to.

Goals of Economic Models

The goals of economic models can be summarized as follows:

1. Economic models are used as tools in the forecasting process, where numerical estimates of the constants of economic relationships are used in terms of the values of independent (explanatory) variables to predict future values of economic volumes (the dependent variables used.
2. Economic models are used to evaluate existing or proposed economic policies. Policy evaluation in economic models provides us with numerical estimates of the constants of economic relationships, which can be used to evaluate existing or proposed economic policies and make decisions about them.
3. Economic models are used in the process of analyzing the economic structure, using appropriate statistical tools [1].

Components of the Economic Mathematical Model:

The economic model consists of the statement of economic theory expressed in economic relationships. This relationship is called a function. For example, it states that consumption (C) is a function of income (y), i.e., $C = F(Y)$, and is called a functional relationship. When we express this function mathematically as an equation, the statement of the above theory is as follows:

$C = Co + C1 (Y)$, assuming the model is linear.

Where:

C: Dependent variable

y: Independent variable, also called the explanatory variable

C0: Represents the constant term, the regression constant

C1: Called the regression coefficient or slope, it represents the magnitude of the effect or change in the dependent variable resulting from a unit change in the independent variable. In this function, it refers to the marginal propensity to consume (MPC). The functional relationship indicates that the dependent variable depends on the independent variables, and the sign does not mean equality here. The symbol f indicates that the relationship is a functional relationship, and the functional relationship indicates that any change in the independent variable or variables leads to a change in the dependent variable [2]. This relationship is written in the form of a linear equation. The model may contain a single equation, in which case it is called a simple equation model, or it may contain several equations, such as the market settlement model or the Keynes macroeconomic model, in which case it is referred to as a simultaneous model or a simultaneous equations model. The equations that make up the simultaneous model are called structural equations. Through this, the model is described, including identifying the economic variables that enter into the composition of the equations contained in the model. It also includes identifying the mathematical form that these equations take, in terms of whether they are linear or non-linear, as this form affects the estimates obtained for the parameters. Therefore, it is necessary to try to determine the appropriate form for the economic relationship, especially since economic theory does not provide us with sufficient information about the nature of the functions and the mathematical form of these functions. The form of dispersion can be used to determine the form of the relationship between two variables and to know whether it represents a straight line or a curve. On the other hand, we can resort to trying different formulas on the data to test the best of them using appropriate statistical criteria in addition to theoretical justifications [3].

Therefore, if there is a relationship between two variables, X and Y, this relationship can take one of the following forms [4].

First: A linear relationship, i.e.,

$$Yi = B1 + B2\ Xi \qquad (1)$$

This equation represents the appropriate form of the relationship between the dependent variable Y and the explanatory variable X, which can be estimated using the least squares method, which is considered one of the most important measurement methods used to estimate the parameters of linear relationships.

Second: The linear relationship between Y and the reciprocal of X,

i.e.

$$yi = B1 + B2\ (1/x) \qquad (2)$$

This equation can be converted to the linear form (1) by performing the transformation $X^* = 1/x$ Therefore, Equation (2) can be reformulated as follows:

$$yi = B1 + B2\ X^*$$

Third: The nonlinear (second-order) relationship between Y and X, i.e.

$$Yi = B1 + B2\ Xi + B3\ Xi^2 \qquad (3)$$

This equation can be converted to the linear form in two variables if we introduce the two variables, X1 and X2, so that

$$Xi = X1i$$

$Xi^{2} = X2i$

In this case, Equation (3) can be written as follows:

$Yi = B1 + B2\ X1i + B3\ X2i$ (3)

Note that the parameters of Equation (3) are the same as the parameters of the original equation (3).

Fourth: The exponential relationship between the variables X and Y, i.e. y = B1 X I B2 (5)

This equation can be converted to linear form by taking the logarithm of both sides as follows:

$Ln\ yi = \ln B1 + B2 \ln Xi$

Or

$Yi = \alpha + B2\ X\ i$ (5)

$Y^* = \ln y$

$X^* = \ln X$

$= \ln B1\ \alpha$

This means that the exponential equation (5) can be converted to a linear equation in the logarithms of the two variables, as in equation (5) = ln B1 α

… B1 = e α = Antilogy (α)

Fifth: The semi-logarithmic relationship between the variables y, x is in the form

$Yi = B1 + B2 \ln Xi$ (6)

Or $Yi = B_1 + B_2 \ln Xi$ (7)

Equations (6) and (7) can be converted to linear form (3) by placing

$X^* = \ln X$ in equation (6) $y^* = \ln y$ in equation (7), and we obtain the following: :

$Yi = B1 + B2\ X^*i$

$Y^* = B1 + B2\ Xi$

The equations included in an economic model are called structural equations (as mentioned previously), given that these equations represent the basic structure of the economics of the facility or industry under study, or of the economy in general. The number of equations varies from one model to another, depending on the ease or difficulty of interpreting the economic phenomenon under study and the objectives the researcher seeks to achieve in formulating the economic model.

The structural equations of the mathematical economic model consist of the following model [5]:

A model consisting of several types of equations, namely:

$Y = C + I + G$ ……………………… (1)

$C = Bo + B1y$.....................................(2)

$T = t1 + t2Y$ ....(3)

$I = g1 + g2\ T$.....................................(4)

$G = \bar{G}$ ……………………….……(5)

1.1 Behavioral Equations.

This is the type of equation that demonstrates the functional relationship between different variables. This relationship arises primarily from a specific economic behavior on the part of individuals or on the part of various elements (producers and investors) that influence the function and show their reactions as a result of changes in certain variables. An example of behavioral equations that appear in the demand function is a functional relationship linking price and income on the one hand, and quantity demanded on the other. Such a function shows us consumers' reactions to changes in price and income, and this reaction appears in the form of an increase or decrease in quantity demanded. Therefore, the first demand equation is considered a behavioral equation [6]. Consumption also responds to changes in income, i.e.:

$Qd = b + dp$….. (1)

$C = Bo + B1$ ….. (2)

The second equation in the model represents the consumption function, which is a behavioral or structural equation that describes consumer behavior. It demonstrates the dependence of C on Y. The slope of the equation is B1, which represents the marginal propensity to consume, and is positive due to the direct relationship between consumer spending as the dependent variable and income level as the independent variable. That is, if income increases, there is an increase in consumption. However, not all of the increase in income goes to consumption; rather, a portion of it goes to savings. Similarly, if we talk about the savings equation, it essentially means that savings (S) is a function of income, and savings change according to changes in income as the independent element and by virtue of the marginal propensity to consume (MPS), which is represented here by (1-b), as in the equation:

$S=- a + (1-b)\, y$…………. (3)

Here, too, we find that the supply function shows a functional relationship linking the quantity supplied of a good to the quantity supplied. On the one hand, and its price on the other hand, meaning that this function shows the producers’ reaction to the change in price, and this reaction appears in the form of changes in the quantity of the commodity that they offer. Therefore, the supply equation is considered a behavioral equation [7].

1.2 Definitional Equations.

They define one variable unconditionally and yield a single, always true, result. These are accounting equations, for example, $Y = C + I + G$. This means that national income equals the sum of the values of the three variables: consumption, investment, and government spending, as in Equation (1). Therefore, income equals consumption and savings. Therefore, we can conclude that savings equals income less consumption [8].

For example,

$S = I$ ... $Y = C + S$………. (1)

So $S = Y - C$ so $I = Y - C$

$Yd = YP - T$

These equations are often represented by identities. An example of such equations is the general budget identity (PB), where:

$PB = TR - TE$

This means that the general budget is defined by the difference between general revenues (TE). It merely reflects a specific fact, without considering the changes in the relevant variables. The identifying equation is also called an identity, meaning it is given. It is often used in market models to represent the relationship between the quantity supplied and the quantity demanded at the equilibrium point. The identifying equation in this case is:

$Qd = Q\,s$

This equation shows us the change that occurs in the quantity supplied of a particular good if the quantity demanded changes, or vice versa.

Although these equations do not explain a specific behavior or clarify the change between variables, the econometric analyst resorts to them in his logical analysis and to complete the economic model [9].

1.3 Technical Equations.

These are equations that explain the technical relationship between the production factors required to complete the production process. These equations illustrate the different ratios of the production factors used within a production unit to achieve a specific volume of production.

Technicians responsible for the production process within the facility determine the production requirements for production factors and determine the required ratios of each factor under the prevailing production conditions, such as:

1. Technical conditions: i.e., the technical level of the equipment and machinery used, as well as the level of expertise of the project's workforce.
2. Existing conditions for financing the project: These conditions determine the project's potential for expansion, whether in terms of capital or production volume.
3. The possibility of obtaining production factors in the quantities and at the level required to complete the production process, and the possibility of substituting one or more of these factors for others.

From this, it becomes clear that the variables included in the technical equations, as well as the coefficients of these variables, are determined by the technicians supervising the production process. The econometric analyst has no role in determining or specifying them. Rather, his role is limited solely to formulating the equation and choosing the formula that meets both the technical and economic conditions, in light of the other equations included in the model [10]. An example is the Cobb-Douglas production function:

$Y = F(K, L)$

$Y = AK^{\alpha} L^{1-\alpha}$

That is, the output Y is a function of the factors of production represented by labor (L) and capital (K), as the two primary factors of production. A is the parameter greater than zero, which measures the productivity of the available technology. We can consider the marginal products of this production function, where the marginal product with respect to labor

$MPL = (1-\alpha) AK^{\alpha} L^{1-\alpha}$

The marginal product of capital is:

$MPL = AK^{\alpha - 1} L^{1 - \alpha}$

Remember that α is between zero and 1. We can write the marginal product of the two factors of production, respectively, as follows:

$MPL = (1 - \alpha)\, Y/L$

$MPK = \alpha\, Y/L$

MPL is proportional to the product per worker, and MPK is proportional to the product per unit of capital. y/L is called the average productivity of labor, and y/K is called the average productivity of capital. If the production function is Cobb–Douglas, the marginal productivity of a worker is proportional to the average productivity.

We can now verify that if workers earn their marginal product, this parameter truly tells us how much income goes to labor and how much goes to capital. The total amount paid to labor, which we saw as.

MPL × L, is equal to Y (-1). Thus, (- 1) is the share of labor in output. Similarly, the total amount paid for capital, MPK × K, equals Y, which is the capital share in output. The ratio of labor income to capital income is constant / (- 1), just as Douglas noted. Factor shares depend only on the parameter, and not on the quantities of capital or labor or on the state of technology as measured by the parameter A.

The Relationship Between Mathematical Economics and Econometrics:

Recently, there has been a significant increase in the use of mathematical methods in most branches of economics, whether in the field of micro- or macro-analysis, for the purpose of clarification or analysis. Traditional economic theory has been reformulated in a mathematical form, enabling the verification of results previously reached by classical economists, strengthening or refuting them, and the derivation of new results that would have been difficult to achieve without the use of mathematics.

It can be said that various economic problems and the economic behavior of individuals and groups, no matter how diverse and varied, are ultimately united by a single characteristic or nature: the problem of maximization. The consumer attempts to maximize the utility or satisfaction they obtain from a particular commodity from among a group of commodities, given their income and prevailing market prices. The producer attempts to maximize their profits from a particular product, obtain the largest possible output at a given cost, or produce a particular output at the lowest possible cost. Thus, it becomes clear to us that utility, output, and costs are merely functions or relationships that link economic variables, some of which are dependent, while others are followed or independent. Whether we express these relationships algebraically, geometrically, or verbally, the problem we ultimately face is the problem of finding the maximum or minimum of a function. Such problems are, in essence, at the heart of mathematics. We also note that economics, although concerned with studying behavioral problems (as mentioned earlier), can be summarized in three types of human activity: consumption, production, and distribution. However, these activities ultimately appear in the form of quantities that the economist is interested in, considering them an expression of this behavior or a result of it. An increase in the quantity demanded of a commodity expresses a change in consumer behavior, and a rise in the price of that commodity is a result of this behavior. An increase in the quantity supplied of a commodity expresses an abundance of production factors or a change in the production technique, and a decrease in its price is a result of this behavior. Here, we see a change in individual behavior manifested in the form of a change in certain economic quantities. Since economics deals with quantities, mathematics becomes a natural means of dealing with these quantities in terms of describing them and finding relationships between them [11].

Therefore, the importance of using mathematical economics in economic analysis lies in the following [12]:

1. The use of mathematical tools in their various forms (arithmetic, algebraic, and geometric) is a useful means of clarification for situations that words cannot fully describe.
2. Formulating economic theories in a mathematical form adds a degree of solidity to assumptions and definitions, making them clear and explicit at every stage of analysis.

For example, we say that $y = a + bx$

Where y = consumption x = income

We understand from this that consumption consists of two parts: one part depends on income, represented by the quantity bx, and the second part depends on factors other than income, represented by the quantity a. We also understand from this expression that we are assuming a constant marginal propensity to consume, represented by the quantity b, while we are assuming that the average propensity to consume is not constant, represented by the quantity + b [13]

3. It is well known that economic variables are numerous and diverse, and the relationships between these variables are intertwined and interconnected. When addressing a problem, we face two difficulties:

First, choosing the variables that are explicitly and clearly included in the model under study and excluding those whose impact is limited or unclear.

Second, the method or approach by which we address the relationships between all or some of these variables. Here, mathematical economics enters the economic field in broader ways a mathematical model [14].

serves as a means of drawing a clear and simplified picture of the real world, which is full of complexity and complexity. It serves as a means of highlighting the basic lines that connect the variables, and of selecting and defining them clearly and precisely.

After expressing a problem with a mathematical model, it is then easier to address it and arrive at clear results. Mathematical economics provides economics with natural tools to address situations that may be difficult to solve using traditional methods [15].

2. Methods

The methodology adopted in this study is grounded in a qualitative, analytical approach that systematically explores the theoretical frameworks and mathematical constructs used in economic modeling. Drawing upon an extensive review of existing literature and established economic theories, this research examines how different types of equations—behavioral, definitional, and technical—function within the broader context of mathematical economics. The investigation centers on the symbolic representation of economic relationships using mathematical equations and functions, such as linear, nonlinear, reciprocal, and logarithmic forms. Emphasis is placed on evaluating the mathematical validity and economic interpretability of models including consumption functions, savings and investment functions, and production models like the Cobb-Douglas function. These models are analyzed based on their structural composition, assumptions, and capacity to reflect real-world economic phenomena. Data used in the discussion are theoretical in nature and drawn from classical macroeconomic frameworks and econometric examples, rather than empirical datasets. Equations were evaluated using logical consistency, theoretical soundness, and their ability to abstractly represent the interaction of variables within an economy. The methodological process also includes comparative analysis to show how variations in mathematical formulation affect the explanatory and predictive strength of economic models. Moreover, the study applies deductive reasoning to assess the role of mathematical transformations in linearizing complex economic relationships, thereby facilitating econometric estimation. The methodology serves not only to classify the types of equations used in economic analysis but also to underscore their functional significance in enhancing clarity, precision, and objectivity in economic modeling.

3. Results

We conclude from the above that mathematical economics presents economic theory in mathematical formulas, namely equations, which take various functional forms. There is no fundamental difference between economic theory and mathematical economics, as each presents economic relationships in a specific (exact) form, such as the consumption function, which expresses Consumption as a function of income, i.e.,

$$Y = f(X)$$

The mathematical formula for this function is:

$$Y = a + bX$$

Therefore, economics mathematical models study quantitative relationships in their abstract form. It is concerned with studying economic theory in its symbolic form.

4. Discussion

The exploration of mathematical economics in this study highlights its indispensable role in modern economic analysis. Through the classification and examination of behavioral, definitional, and technical equations, the discussion reveals that mathematical modeling serves as a vital bridge between abstract economic theory and its practical application. Behavioral equations, for instance, reflect real-world economic behavior by establishing functional relationships between variables such as income and consumption or price and demand. These equations provide insights into how individuals and firms respond to economic stimuli, thus enabling more accurate predictions and evaluations of policy outcomes.

Definitional equations, while less dynamic in nature, play a foundational role in structuring models by establishing identities—such as the national income equation—that ensure internal consistency. Their use affirms that not all economic relationships are driven by behavioral change but may instead serve to anchor models in accounting logic. Technical equations, on the other hand, reflect production processes and technological constraints, as demonstrated in the Cobb-Douglas function. These equations illustrate the efficient allocation of labor and capital inputs and help quantify productivity under varying economic conditions.

A key implication discussed is the flexibility of mathematical models to adapt to different functional forms—linear, nonlinear, exponential, and logarithmic—each offering unique advantages in modeling economic complexity. The transformation of non-linear relationships into linear forms is particularly crucial for econometric estimation, enhancing analytical tractability and statistical robustness.

Overall, the discussion underscores that mathematical economics, by abstracting and simplifying economic relationships, provides a rigorous and coherent framework for understanding the economy. It also paves the way for integrating economic theory with econometric analysis, contributing to more evidence-based decision-making. Further research may explore empirical validation of these models in specific economic contexts, such as emerging markets, to assess their practical relevance and predictive accuracy.

5. Conclusion

In conclusion, this study affirms the centrality of mathematical modeling in economic theory, demonstrating that the use of equations—behavioral, definitional, and technical—provides a structured and precise means of expressing complex economic relationships. The key finding is that mathematical economics not only simplifies theoretical constructs but also enhances analytical clarity and predictive accuracy by employing various functional forms, including linear, nonlinear, and logarithmic models. These models allow for the abstraction of economic behavior and the construction of theoretical frameworks that are essential for policy analysis, forecasting, and structural evaluation. The implications of this research lie in the reinforcement of mathematical economics as a foundational pillar for integrating theory with econometric analysis, ultimately enabling more robust and objective interpretations of economic dynamics. Future research should aim to empirically validate the theoretical models discussed, particularly in the context of developing economies, by applying real-world data to assess the effectiveness and adaptability of mathematical frameworks in diverse socio-economic environments.

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