Innovative: International Multi-disciplinary Journal of Applied Technology
(ISSN 2995-486X) VOLUME 03 ISSUE 06, 2025

GRAPH MODELS IN CALCULATING TIME CHARACTERISTICS AND CRITICAL PATH

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Abstract:
This paper examines graph models used for scheduling a set of interrelated tasks. The focus is on methods for calculating the time characteristics of activities, such as early and late start and finish dates, and determining the critical path of a project - the sequence of activities that determines the minimum duration for completing all work. The construction of a network graph reflecting the logical structure of the project and algorithms for analyzing time parameters based on it are considered. The role of computer technologies in automating calculations and visualizing network models is also emphasized, which contributes to more efficient project and resource management. The work may be useful for students, project management specialists and those interested in methods of planning and optimizing work processes.

Keywords: Network Schedule, Critical Time, Operations, Critical Path, Time Parameters, Reserve Interval, Event

1. Introduction

With the emergence of complex interrelated operations in production and economics, the importance of developing more accurate and effective methods for planning the consumption of time and resources has increased. Based on the results of calculating the time parameters of the network schedule, a calendar schedule (plan) for a set of works is drawn up. When drawing up a calendar plan, it is necessary to take into account the volume of available resources, since due to limitations in labor, tools, equipment and other resources, it is impossible to perform some operations simultaneously. In this regard, regular stocks for non-critical operations are also important. By changing the duration of non-critical operations by a certain amount, without exceeding their standard stock, it is possible to reduce the maximum need for resources [1]. A set of operations is a set of interconnected operations aimed at achieving a specific goal, the implementation of which must occur in a specific sequence. In this case, the operations are logically linked in such a way that the beginning of one operation requires the completion of several others.

Each operation in a complex, as a rule, represents a certain work, the execution of which requires time and resources. The main task of planning a complex of operations is to determine the start and end dates of each operation, as well as identifying operations that have a decisive influence on the overall execution time of the entire complex. The construction of a network model is the first step in the implementation of network planning and management of a complex of operations. Based on the constructed network schedule, a calendar plan for the implementation of a complex of operations is created. The construction of a calendar plan is associated with the time parameters of the network schedule, the concepts of a critical path, a critical event, and a critical operation [2, 3,4].

In a network diagram, the length of a path is the total time required to complete all the activities that fall on that path. The longest complete path is called the critical path. There can be more than one critical path in a network. In a network diagram, the critical path is usually represented by a thick line. The length of the critical path is called the critical time. Critical time is the time required to completely complete all the operations in a complex. Operations and events that occur on the critical path are called critical operations and critical events. We denote the critical path as $\mu_{кр}$, and the critical time as $t_{кр}$.

The time parameters of the network schedule include the total time of execution of a set of operations (critical time), early and late dates of occurrence of each event, early and late dates of start and end of each operation, as well as time reserves of events and operations.

Let tij denote the duration of each operation Pij in the network graph. Let U - be the set of all arcs (operations) in the network. Let j denote the set of arcs entering the j-th U_j^- node, and let i denote the set of arcs leaving the i -th U_i^+ node.

From the definition of the quantity ti it is easy to see that,

$$t_1 = 0, \quad t_j = \max_{(i,j)\in U_j^-}(t_i + t_{ij}), \quad j = 2,3,\ldots,n \quad , \tag{1}$$

this formula is valid.

Based on the definition of the critical path and formula (1), the following conclusions can be made:

a) ti is the maximum length of the paths preceding the i -th event in the network diagram;

b) the part of the critical path from the initial event to the i -th critical event is the longest path along the i -th event.

From these conclusions it follows that the critical path is the $1, i_1, i_2, \ldots, i_k, n$ early time of occurrence of events.

$$t_1 \le t_{i_1} \le t_{i_2} \le \ldots \le t_{i_k} \le t_n \, , \tag{2}$$

$$t_{i_1} - t_1 = t_{1i_1}, \; t_{i_2} - t_{i_1} = t_{i_1 i_2}, \; \ldots, \; t_n - t_{i_k} = t_{i_k n} \tag{3}$$

satisfying relationships.

Thus, if the early times of $t_1, t_{i_1}, t_{i_2}, \ldots, t_{i_k}, t_n$ occurrence of events are arranged in ascending order, and a sequence is formed consisting of the corresponding vertices and arcs of the network graph, which forms a path $\mu = (1 \to i_1 \to i_2 \to \ldots \to i_k \to n)$, when relations (2) and (3) are satisfied, then such a path μ is a critical path [4, 5].

The early start time of the i-th event is also the early start time of each operation Rij that results from this event. We denote the early start time of operation Rij as $t_{ij}^{э.т.}$.
Then:

$$t_i = t_{ij}^{э.б.}, \ \forall (i,j) \in U_i^+ \qquad (4)$$

The early completion time of the operation Rij is a value that is determined by the formula:

$$t_{ij}^{э.т.} = t_i + t_{ij}, \ \forall (i,j) \in U_i^+ \qquad (5)$$

where $t_{ij}^{э.т.}$ — duration of operation Rij.

Quantities ti, $t_{ij}^{э.т.}$, $t_{ij}^{э.б.}$ are defined for an arbitrary i-th event and each operation Rij emanating from this event.

If event i is a critical event, then the part of the critical path from this event to the final event is also critical, that is, it is the longest path in time among the paths after event i. Therefore, it is impossible to postpone the early start time of critical operations. Consequently, each critical operation Rij is the operation that must be executed first when the expected time ti of event i occurs. Therefore, it is impossible to postpone the early start time of critical events. Early start dates of non-critical operations and events may be postponed for a certain period of time. Such delays, being within a certain interval (boundary), do not affect the total execution time of the complex of operations. Let us define the threshold delay time (late onset time) for the occurrence of the non-critical i-th event as t_i^*.

The time (duration) of the i-th event is determined $t_i^* = t_n -$ equality {maximum path length in time after the i-th event}. From the definition

$$t_i^* = \min_{(i,j)\in U_i^+} (t_j^* - t_{ij}), \quad i = 2,3,\ldots,n-1, \quad t_n^* = t_n, \quad t_1^* = t_1 = 0 \qquad (6)$$

relations follow.

It should be noted that $t_i = t_i^*$ equality holds for each i-critical event. We use this property as a sufficient condition in addition to the necessary conditions (2) and (3) in defining the critical path, critical events, and critical operations. The late occurrence time of events is closely related to quantities called the late start time and late finish time of the operations that result from the event [6, 8]. Let us denote later $t_{ij}^{к.б.}$ start time and later $t_{ij}^{к.т.}$ the end time of the operation Rij, occurring from the i-th event, as These quantities are defined as follows:

$$t_{ij}^{к.б.} = t_j^* - t_{ij}, \quad t_{ij}^{к.т.} = t_j^*. \qquad (7)$$

It is important to note that the early and late start times of a critical activity are the same thing - the expected start time of the critical activity. Similarly, the early and late finish times of a critical activity are the same quantity. Now we will provide information about the time reserves of events and operations. The time reserve of the i-th event is called the value Ri, which is determined by the formula $R_i = t_i^* - t_i$.

The time interval is $[t_i, t_i^*]$ called the freedom time interval or the reserve interval of the i-th event.

The occurrence of an event at a time corresponding to a freedom interval does not change the time required to completely complete all the tasks in the set of operations.

For each operation Rij, it is interesting to know how long event j can be delayed without violating the expected time of its occurrence.

This is the possible value of the delay in the start time of the operation Rij, which is called the free reserve time of the operation Rij, and is expressed by the value determined by the formula

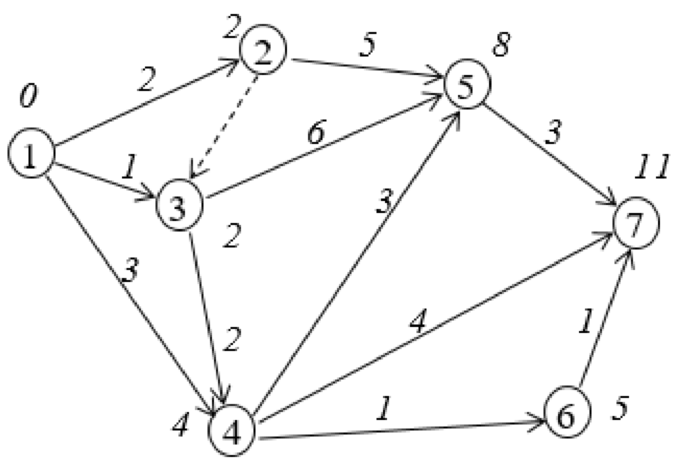
$$R_{ij}^{э} = t_j - t_i - t_{ij} = t_j - t_{ij}^{э.m.} \quad (8)$$

The total reserve time of the operation Rij, is the value calculated using the formula:

$$R_{ij}^{m} = t_j^{*} - t_i - t_{ij} = t_{ij}^{к.m.} - t_{ij}^{э.m.} \quad (9)$$

2. Statement of The Problem

A set of operations is a collection of interconnected operations aimed at achieving a specific goal, the implementation of which requires compliance with a certain order. The main task of planning a complex of operations is to determine the start and end dates of each operation, as well as to identify operations that have a decisive influence on the overall duration of the complex. Let us consider the calculation of time parameters of events and operations, as well as the critical path, using the example of the network graph below.



Given a network graph, which is a model of a project consisting of 10 events (vertices) and a set of operations (arcs), each of which is characterized by its execution duration.

Operations are directed from one event (beginning) to another (end) and reflect the logical sequence of execution of a set of works.

Perform a project analysis using a graph model to determine:

1. Early event dates;
2. Late event dates;
3. Operation slack;
4. The project's critical path - the sequence of operations that determines the minimum possible completion time for the entire project.

Initial data:

The network graph is presented as a directed acyclic graph, where:

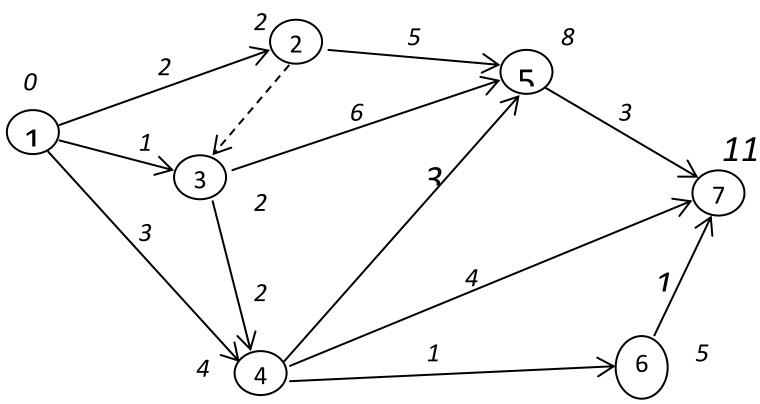
• Nodes denote events (start or end of operations),

• Edges (arcs) — operations with a given duration.

Required:
1. Calculate:
o Early event dates (by forward pass method),
o Late event dates (by backward pass method),
2. Determine:
o Total float for each operation,
o Partial (free) float, if necessary,
3. Find:
o Critical path — the path through operations with zero total float,
o Total project duration.

3. Methods of Solving The Problem, Auxiliary Results

Let us consider the calculation of time parameters of events and operations, as well as the determination of the critical path using the example of the given network graph.



First we find the expected (early) time of occurrence of $t_i,\ i=\overline{1,7},$ events. It will be as $t_1=0$ agreed. Only operation R_{12} leads to event 2, and since the execution time of this operation is $t_{12}=2$, the early time of occurrence of event 2 is $t_2=t_1+t_{12}=0+2=2$.

There are operations R_{13} and R_{23} that lead to event 3. Therefore, $U_3^-=\{(1,3),(2,3)\}.\ t_{23}=0.$ since R_{23} is a pseudo-operation, then according to the formula (1) $t_3=\max\{t_1+t_{13};t_2+t_{23}\}=\max\{0+1;2+0\}=2$

Event 4 includes two operations R_{14} and R_{34}, which are derived from events 1 and 3, that is, that is, because $U_4^-=\{(1,4),(3,4)\}$, according to formula (1), we get $t_4=\max\{t_1+t_{14};t_3+t_{34}\}=\max\{0+3;2+2\}=4$, that event 4 is an early time of occurrence.

We calculate the values in a similar way $t_5,=8,\ t_6,=5,\ t_7=11$. Let us write down the found early times of occurrence of t_i events on the graph (Figure 1) next to the circle representing each i-th event.

So, , $t_1=0,\ t_2=2,\ t_3=2,\ t_4=4,\ t_5=8,\ t_6=5,\ t_7=11$. Critical time $t_{kr}=t_7=11$.

Now, using formula (6), we determine the number of limiting (late) times $t_i^*,\ i=\overline{1,7},$ occurrence of events.

Since for $U_5^+=\{(5,7)\},\ U_6^+=\{(6,7)\}$ events 5,6, according to the formula (6)

$t_5^* = t_7^* - t_{57} = t_7 - t_{57} = 11 - 8 = 8,\ t_6^* = t_7^* - t_{67} = t_7 - t_{67} = 11 - 1 = 10.$

Since this is for $U_4^+ = \{(4,5),(4,6),(4,7)\}$ events 4, then according to the formula (3.6)

$t_4^* = \min\{t_5^* - t_{45}, t_6^* - t_{46}, t_7^* - t_{47}\} = \ = \min\{8-3, 10-1, 11-4\} = \min\{5,9,7\} = 5$

we find it. [6, 7].

Continuing this way, $t_3^* = 2,\ t_2^* = 2$ we find the values. It will be like this, $t_1^* = t_1 = 0$ as decided. And so, $t_1 = t_1^*$, $t_2 = t_2^*$, $t_3 = t_3^*$, $t_5 = t_5^*$, $t_7 = t_7^*$,

$t_2 - t_1 = t_{12}$, $t_3 - t_2 = t_{23}$, $t_5 - t_3 = t_{35}$, $t_7 - t_5 = t_{57}$,

relationships are adequate.

It shows, $\mu_{кр} = (1 \to 2 \to 3 \to 5 \to 7)$ that this is the critical path.

Based on the data obtained, it is easy to determine the early and late start times, as well as the early and late finish times of all operations. How much P_{12}, P_{23}, P_{35}, P_{57} яв are critical operations,

$t_{12}^{э.б.} = t_{12}^{к.б.} = t_1 = 0$, $t_{23}^{э.б.} = t_{23}^{к.б.} = t_2 = 2$, $t_{35}^{э.б.} = t_{35}^{к.б.} = t_3 = 2$,

$t_{57}^{э.б.} = t_{57}^{к.б.} = t_5 = 8$, $t_{12}^{э.т.} = t_{12}^{к.т.} = t_2 = 2$, $t_{23}^{э.т.} = t_{23}^{к.т.} = t_3 = 2$,

$t_{35}^{э.т.} = t_{35}^{к.т.} = t_5 = 8$, $t_{57}^{э.т.} = t_{57}^{к.т.} = t_7 = 11$.

For critical events, the length of the reserve interval is zero. Since the values of total and free reserve time for critical operations are zero.

$$R_{12}^{т} = R_{12}^{э} = R_{23}^{т} = R_{23}^{э} = R_{35}^{т} = R_{35}^{э} = R_{57}^{т} = R_{57}^{э} = 0$$

We calculate the reserve interval, as well as reserves of full and free time for non-critical events and operations.

Events 4–6 are not critical. The reserve intervals for them are, respectively, $[t_4, t_4^*] = [4, 5]$, $[t_6, t_6^*] = [5, 10]$.

For non-critical operations P_{13}, P_{14}, P_{34}, P_{45}, P_{46}, P_{47}, P_{67} calculate the values $t_{ij}^{э.б.}$, $t_{ij}^{э.т.}$, $t_{ij}^{к.б.}$, $t_{ij}^{к.т.}$ according to formulas (4), (5), (7).

For example, for P_{13}

$t_{13}^{э.б.} = t_1 = 0$, $t_{13}^{э.т.} = t_1 + t_{13} = 0 + 1 = 1$,

$t_{13}^{к.б.} = t_3^* - t_{13} = t_3 - t_{13} = 2 - 1 = 1$, $t_{13}^{к.т.} = t_3^* = t_3 = 2$.

and for P_{34}

$t_{34}^{э.б.} = t_3 = 2$, $t_{34}^{э.т.} = t_3 + t_{34} = 2 + 2 = 4$,

$t_{34}^{к.б.} = t_4^* - t_{34} = 5 - 2 = 3$, $t_{34}^{к.т.} = t_4^* = 5$

will.

We will calculate the total and free time reserves for non-critical operations using formulas (8), (9).

For example, for operations P_{13}, P_{34}

$R_{13}^{т.} = t_3^* - t_1 - t_{13} = t_3 - t_1 - t_{13} = R_{13}^{э.} = 2 - 0 - 1 = 1$,

$$R_{34}^{т.} = t_4^* - t_3 - t_{34} = 5 - 2 - 2 = 1,$$
$$R_{34}^{э.} = t_4 - t_3 - t_{34} = 4 - 2 - 2 = 0.$$

4. Computational Experiment

The calculations performed are presented in Table 1.

Table 1. Calculations performed.

Operation (*i,j*)	Operation duration t_{ij}	Early dates		Late dates		Full reserve	Free reserve
		Start $t_{ij}^{э.б.}$	End $t_{ij}^{э.т.}$	Start $t_{ij}^{к.б.}$	End $t_{ij}^{к.т.}$	R_{ij}^{m}	$R_{ij}^{э}$
(1,2)	2	0	2	0	2	0	0
(1,3)	1	0	1	1	2	1	1
(1,4)	3	0	3	2	5	2	1
(2,3)	0	2	2	2	2	0	0
(3,4)	2	2	4	3	5	1	0
(2,5)	5	2	7	3	8	1	1
(3,5)	6	2	8	2	8	0	0
(4,5)	3	4	7	5	8	1	1
(4,6)	1	4	5	9	10	5	0
(4,7)	4	4	8	7	11	3	3
(5,7)	3	8	11	8	11	0	0
(6,7)	1	5	6	10	11	5	5

As can be seen from the table, only critical operations have zero total and free time reserves. Non-critical operations may also have no free time reserve. For example, for P_{34} and P_{46} non-critical operations, the free time reserves are zero[8], [9].

5. Discussion of Results and Conclusion

The necessary algorithms for implementing calendar planning on a computer based on a network model of a complex of operations have been developed, and computer programs that ensure the operation of this algorithm have been presented.

The obtained results of calendar planning based on the network model of the production complex can be used to implement optimal decision-making and management in planning issues arising in the economy using computer technology.

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