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Hybrid Waveform Diagnostics of Seasonal Time Series Models and Exponential Smoothing with the Application

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Abstract:

The study aims to evaluate the effectiveness of wavelet analysis as a diagnostic tool for univariate time series models by comparing several traditional and wavelet-based methods. It also seeks to compare the performance of two different approaches for analyzing and forecasting univariate time series: traditional exponential smoothing and wavelet analysis. Additionally, the research explores the effectiveness of a hybrid model combining exponential smoothing and wavelet filters. These methods were applied to real-world data, with results showing that the hybrid model achieves higher predictive accuracy and excels in isolating noise and abrupt changes.

Keywords: Time series, exponential smoothing, wavelets, hybrid model.

1. Introduction:

Time series analysis is one of the most prominent statistical methods, focusing on the study of phenomena by examining their historical development over time—whether daily, weekly, or quarterly—to generate estimates with minimal error. Additionally, time series analysis has become a critical tool for decision-making in applied fields.

Seasonality in time series refers to a recurring pattern of changes that repeats at fixed intervals. It poses a significant challenge for researchers, as improper handling can negatively impact the accuracy of mathematical models (Abu Dahrouj, 2023; Muzahem et al., 2023).

Forecasting is a fundamental technique for predicting future trends in economic institutions. It relies on historical and current data to project future outcomes. Forecasting tools are widely applied in time series analysis, as most data in this context are collected over past periods (Youssef & Tawfiq, 2022).

2. Time Series:

A time series can be defined as a sequence over time. Successive observations are usually independent, meaning they depend on each other. This lack of independence is exploited to arrive at reliable predictions. We will also use the subscript (1) to indicate the temporal order of observations. Thus, (Z_t) represents observation number (t), (Z_{t-1}) represents the previous observation, while (Z_{t+1}) represents the next observation (Jaber, Majeed, 2017)(Muzahem et al.,2025)

It is defined as a random process of historical data collected over time (Mahmoud, 2010), and is described as a statistical time series, which is a set of observations that have occurred over time (Ahmed, Al-Jubaili, 2021).

3. Exponential Smoothing:

Smoothing is defined as the process of smoothing or smoothing data. It is a type of estimation that has proven successful in studying cases that depend on or change over time. Furthermore, smoothing produces highly efficient results, reducing missing values using prediction, or what is known as the (Naive) random walk method, compared to traditional methods such as the simple arithmetic mean and the moving arithmetic mean (Ahmed, Al-Jubaili, 2021) (Mahmoud, 2010).

1.3 Exponential Smoothing Methods for Time Series

Exponential smoothing methods are one of the essential tools in time series analysis, as they are used to predict future values based on historical time series data. These methods are characterized by their simplicity and effectiveness and are used in many fields such as economics, business, engineering, energy, and others. Exponential smoothing methods range from simple methods that rely on past values only to complex methods that take into account trends and seasonality. Exponential smoothing encompasses several key approaches that vary in complexity based on the time series characteristics they address: Simple Exponential Smoothing (SES)،Double Exponential Smoothing (Holt's Method) ،Triple Exponential Smoothing (Holt-Winters Method)

2.3 Seasonal Exponential Smoothing (Holt-Winters):

When the time series exhibits seasonal variation, the Holt model yields inaccurate results. Therefore, the model is adjusted to account for the seasonal component when detected in the series. This is done using the Winters equation, resulting in what's known as the Holt-Winters model, which considers both the overall trend and seasonal variations (Youssef, Tawfiq, 2022).

4 .Wavelet Transformation

This is a mathematical analysis method that processes signals by converting them between the time and frequency domains using a variable-width window rather than a fixed-width window (Nassar, 2023).

Wavelet analysis stands as a crucial tool for analyzing and extracting meaningful information from raw time series data (Jaloul, Asmahan, 2021).

1.4 Wavelets

Wavelets serve as a fundamental tool for signal analysis and data transformation, particularly in signal processing and dynamic systems. This method operates by decomposing signals into multi-frequency, multi-dimensional components, enabling the identification of intricate signal patterns. Its adaptability makes it particularly effective for analyzing non-stationary data and handling noise.

Our discussion will focus on two widely-used wavelet types in analytical applications: the Haar wavelet and the Daubechies wavelet.

1- Haar wavelet:

The Haar wavelet, developed by Alfred Haar in 1915, represents one of the earliest and most fundamental wavelet transform methods. It is widely recognized for its computational simplicity among wavelet techniques (Hassan, Trad, 2019).

2- Daubechies wavelet:

Ingrid Daubechies stands as a preeminent figure in wavelet research.She developed various transform functions named after her (the Daubechies family), each of which is symbolized by (dbN), where (N) represents the function's reference (Al-Muntasir, 2019).

3- Coifelets:

Daubechies designed these filters in response to a suggestion by Coifelet, who needed this structure for use in numerical analysis applications. These filters differ from Daubechies filters in that they contain both negative and positive values (Sarah K. Bleiler, 2008).

5. Criteria for Selecting the Best Model

Criteria for selecting the best model are essential elements in the fields of analysis and forecasting, as choosing the appropriate model significantly impacts the accuracy of the results and the effectiveness of its applications in various fields such as statistics, artificial intelligence, economics, engineering, and others. Selecting the most appropriate model depends on a number of criteria that aim to achieve a balance between the accuracy of predictions, simplicity, and the strength of generalization. These criteria include)(Muzahem et al.,2023):

1. Akaike Information Criterion (AIC)

This criterion was proposed by Akaike in 1973 and is one of the most important criteria for determining the rank of a model. Its mathematical formula (Mohammed, 2014)(Heyam et al.,2025) is- :

$$(AIC)(M) = n\,Ln\,\sigma^2 + 2M \qquad (1)$$

And the model that is best fits the data series will be chosen, corresponding to the lowest value of the AIC criterion (Al-Marshadi, 2021).

2. Corrected Akaike Information Criterion (AIC):

This criterion was proposed by Hurvich and Tsai in 1989 (Hurvich and Tsai, 1989) for use when observations are few. Its mathematical formula is (Al-Jamal, Al-Omari, and Saleh, 2011):

$$CAIC \;=\; n\log\sigma^2 + \frac{2n(m+1)}{n-m-2} \qquad (2)$$

3. Seasonally Modified Akaike's Information Criterion (SMAIC):

The Akaike criterion for the seasonal model performs better and more efficiently than the traditional Akaike criterion in determining the rank of the multiplying seasonal model (Kalilar and Erdemir, 2003). The general formula for this criterion is (Al-Marshadi, 2021):

$$SMAIC \;= n\,Ln\sigma^2 + \left(\frac{2d^2K}{T}\right) + ti \qquad (3)$$

4. Bayesian Information Criterion (BIC)

This criterion was proposed by Schwarz in 1978 to address how to select one model from several models with an unequal number of independent variables by finding their Bayesian solution. It was expanded to a Bayesian solution using Bayesian theorem (Abdul Rahim, Jawad, 2021).

This criterion has the following formula (Al-Marshadi, 2021)- :

$$BIC(P) = n\, In\sigma^2 + P\, In\, n \qquad (4)$$

5. Criteria for trade-offs between models:

Model selection criteria play a crucial role in identifying the optimal solution for a specific problem, as they enable systematic evaluation of competing models based on their distinctive features. These benchmarks allow practitioners to assess and contrast model performance, ultimately choosing the solution that best aligns with both the dataset characteristics and problem objectives. The evaluation framework incorporates multiple technical and practical dimensions that collectively determine a model's operational performance and practical utility (Al-Kalabi, 2018).

1. Mean Squared Error (MSE)

$$MSE = \frac{1}{n}\sum_{t=1}^{n} e_t^2 \qquad (5)$$

2. Mean Absolute Error (MAE)

$$MAE = \frac{n}{1}\sum_{t=1}^{n} |et| \qquad (6)$$

3. Mean Absolute Percentage Error (MAPE)

$$MAPE = \frac{\sum_{t=1}^{n} | \frac{Y_t - \hat{Y}_t}{Y_t}}{n} * 100 \qquad (7)$$

4. Root Mean Squared Error (RMSE)

$$RMSE \sqrt[2]{\frac{1}{n}\sum_{t=1}^{n} e_t^2} \qquad (8)$$

5. Mean Absolute Deviation (MSD)

The mean deviation serves as an accuracy metric for predictions, where smaller values indicate better alignment between estimated and observed values (Mahmoud, Ibrahim, 2022)- :

$$MSD = \frac{1}{N}\sum_{t=1} e_t^2 \qquad (9)$$

6. Hybrid Model:

The hybrid model is an approach that combines different techniques or methods from various fields to achieve optimal results in data processing or predictions. This type of model leverages the advantages of multiple methods to overcome limitations present in each individual model when used separately. Hybrid models are employed across numerous fields including artificial intelligence, machine learning, engineering, economics, and healthcare, where they integrate traditional models with modern methods to enhance performance and produce more accurate and reliable results (Nassar, 2023).

1.7 Reasons for Using a Hybrid Model:

1- Benefiting from the advantages of different models: A model may be strong at handling a certain type of data but weak at handling another. By combining models, multiple advantages can be leveraged to deliver better performance.

2- Improving Predictive Accuracy: A hybrid model can help reduce errors and increase the accuracy of predictions or classifications by combining predictions from several different models. For example, supervised models are sometimes combined with unsupervised models to obtain a more comprehensive analysis.

3- Dealing with Complex Challenges: In many cases, it may not be possible to process complex data using a single model. By combining models, problems such as dynamic changes, noise, and incomplete data can be addressed.

7 .The Applied Part

Humanity has known about petroleum and natural gas for about 5,000 years (1) (Rashid Mahdi Ahmed, "Geography of Petroleum" p. 155). Natural gas contains obstructions, which are invisible to the naked eye (microscopic), such as algae and protozoa that have accumulated over the years in the layers of the earth and oceans. The remains were compressed under sedimentary layers. As a result of pressure and heat, these remains and organic materials were transformed thousands of years later into natural gas. Natural gas is no different in composition from petroleum, as both are created under the same conditions, as they are often found in underground or underwater fields (Dhiab, Nasri, 2011).

In this research, monthly data was obtained representing the monthly quantities of natural gas consumed in the United States, usually measured in cubic feet, for the period from January 1, 2000 to August 1, 2024, with *296* observations. The data was examined according to the *goodness of fit* test, and the data was analyzed using the *R* language.

The first steps in analyzing the data A time series is a plot of observations on the vertical axis and the time element on the horizontal axis.

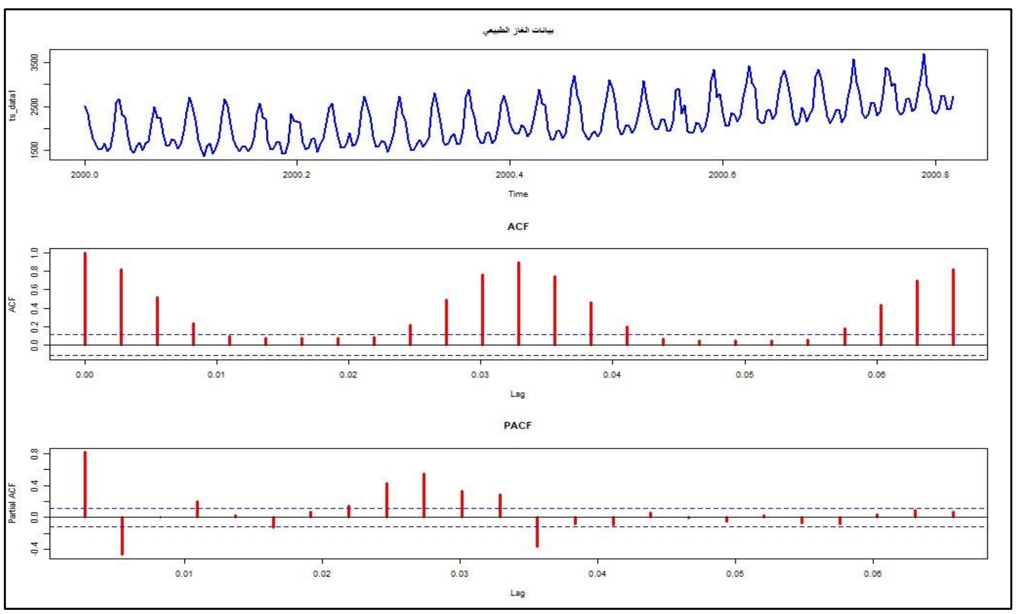

Figure (1): Time series plot representing monthly data on natural gas consumption in the United States, and plotting the autocorrelation and partial autocorrelation functions.

We note from the first plot a general increasing trend over time, as well as the presence of fluctuations that recur regularly and at the same rate every year. This means that the time series is not stationary. To stabilize the series, normal and seasonal differences were used for the time series data. The stationarity of the resulting series was then confirmed using the Dickey-Fuller test. The results obtained are shown in the following table:

Table (1): Results of the Dickey-Fuller test for the series after taking the first difference and seasonal differences.

Test	Test Statistics	Table Statistics	P-value
Augmented Dickey-Fuller	-12.105	-3.43	0.01

Table (1) shows that the absolute value of the test statistic is greater than the table value of the Dickey-Fuller test, and the p-value≅0 is less than the significance level (0.05), which calls for rejecting the null hypothesis:

$$H_0: Non - Stationary\ vs\ H_1: Stationary$$

The seasonal Box-Jenkins methodology was then applied to forecast natural gas consumption in the United States by phase, including the identification and estimation phases. The statistical parameters for this model, namely the Akaky Information Criterion (AIC), the AIC, the Bayesian Criterion (BMC), and the mean square error estimator, can be seen in the following table:

Table 2: Shows the criteria for the best model obtained

Model	AIC	AICc	BIC	MSE
$SARIMA(2,1,1)(2,1,1)_{12}$	12.191	12.192	12.287	13.32

The values of the parameters of the best model obtained were then estimated. The estimated values of the model parameters can be seen in the following table:

Table (3): Estimates of the parameters of the best model obtained

Parameters	Estimate	SE	t-value	p-value
$AR(1)$	0.334	0.003	89.96	0.000
$AR(2)$	0.017	0.005	3.309	0.001
$MA(1)$	-0.900	0.028	-31.39	0.000
$SAR(1)$	0.043	0.005	8.44	0.000
$SAR(2)$	-0.285	0.002	-135.20	0.000
$SAM(1)$	-0.776	0.023	-32.61	0.000

We note from the table that the p-values corresponding to all model parameters are less than the significance level of α = 0.05, which indicates that the model parameters are significant.

To verify the validity of the model, the model residuals were plotted.

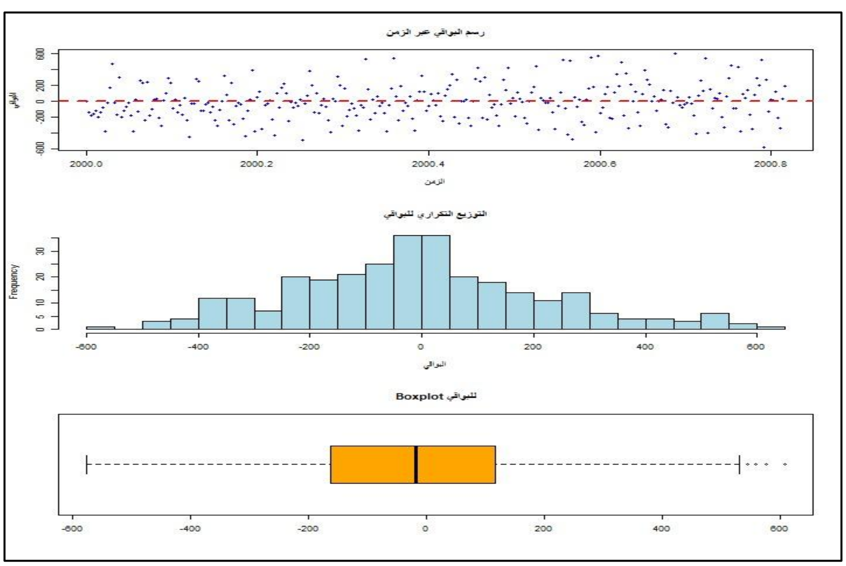

Figure (2): The random plot and frequency distribution, as well as the box plot of the model's residuals, are shown.

We notice from the figure above, which represents the scatter plot of the residuals over time, that the residuals are distributed almost randomly around zero. This indicates the absence of a clear temporal pattern, indicating that the obtained model fits the data.

While the histogram shows a shape close to a normal distribution, it tends to be skewed slightly to the right. This is a positive skew, indicating that the residuals are approximately normal.

The box plot of the model's residuals shows some outliers outside the minimum and maximum, as shown on the right side. Based on the above, we used it to test the selected model and verify its suitability for use in prediction. Several observations were predicted, and the results are illustrated in the following figure:

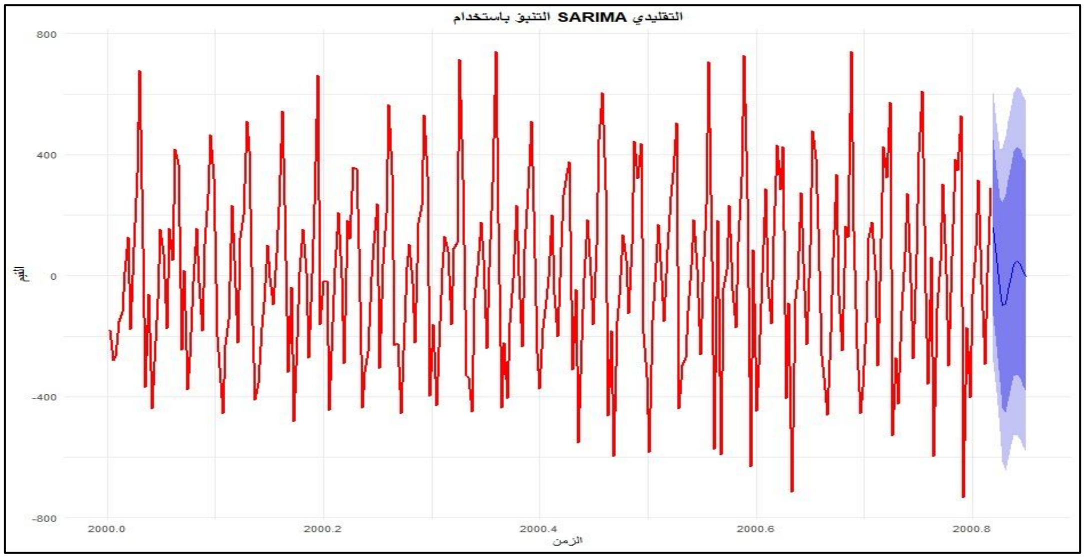

Figure (3): Shows the predictive values of the obtained model.

To determine the extent to which the predictive values obtained from the best model matched the actual data for the selected sample, the predicted values were plotted. It was observed that the actual values matched the predicted values, as shown in the following figure:

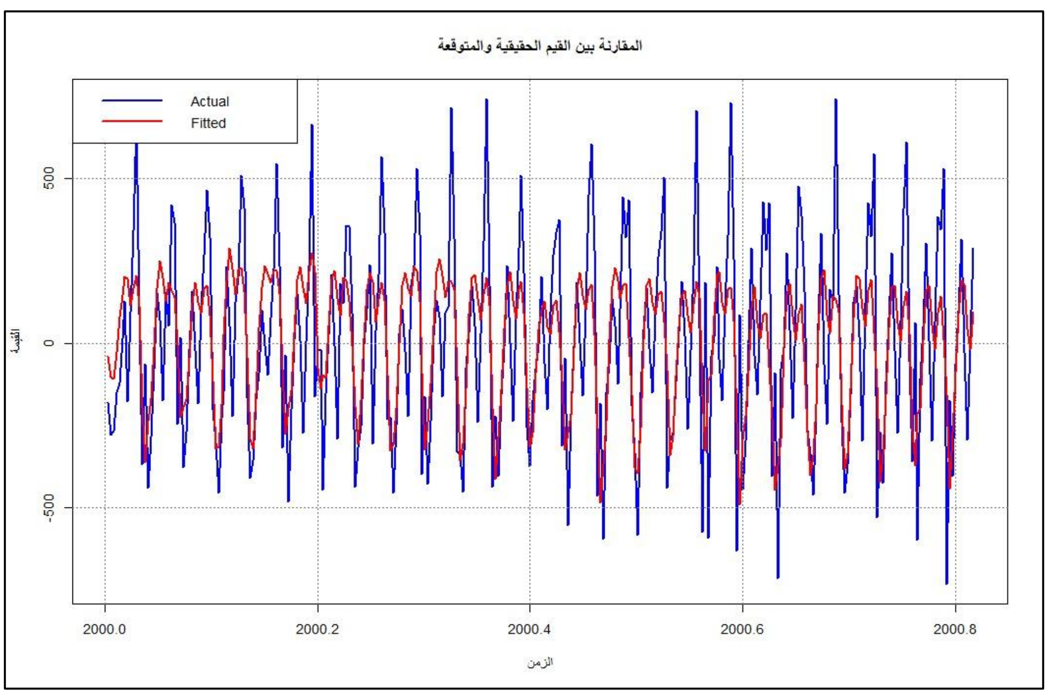

Figure (4): Actual values with predicted values.

Some future predictive values can also be illustrated from September 2024 to April 2025, which is equivalent to eight monthly predictive values, as shown in the following table:

Table (4): Predictive values of the best model obtained.

Date	Predicted values of
September/2024	2423.0
October/2024	2436.3
November/2024	2716.1
December/2024	3361.4
January/2025	3921.9
February/2025	3243.7
March/2025	3572.8
April/2025	3425.0

8.1 Forecasting Using Seasonal Exponential Smoothing (Hybridizing Exponential Smoothing with Wavelet Filters) of Real Data

This step involves taking a set of combinations for each method and the three parameters on the forecast equation for the seasonal time series, using the MSD criterion, which gives the lowest value for the best smoothing and also indicates the correctness of the smoothing.

8.2 Hybridizing Exponential Smoothing with Haar Wavelet Filters for Monthly Natural Gas Consumption Data

When combining and hybridizing wavelet smoothing with exponential smoothing, noise is removed from the data, which helps improve forecasting accuracy. Using the Haar filter for monthly natural gas consumption data to obtain optimal values for the three parameters using the Holt-Winters method is shown in the following table:

Table (5): Forecasting Using the Holt-Winters Method with the Approved Parameter Values Using the Haar Filter for Monthly Natural Gas Consumption Data

Filters	Optimal parameter values			MSD
	α	β	γ	
Haar	0.603	0.004	0.63	32.42

A value of $\alpha = 0.603$ indicates that the series contains some distortion or noise, while a value of $\beta = 0.004$ indicates that the trend effect is somewhat weak. A value of $\gamma = 0.63$ represents a general adjustment factor, or what is called a weight for the wavelet effect. Overall, the Haar filter improved the prediction accuracy with an MSD value of 32.42 and reduced the effect of seasonality and trend. As shown in the forecast figure, which illustrates the prediction of the hybrid Holt-Winters exponential smoothing model with the Haar filter, as shown in the following figure:

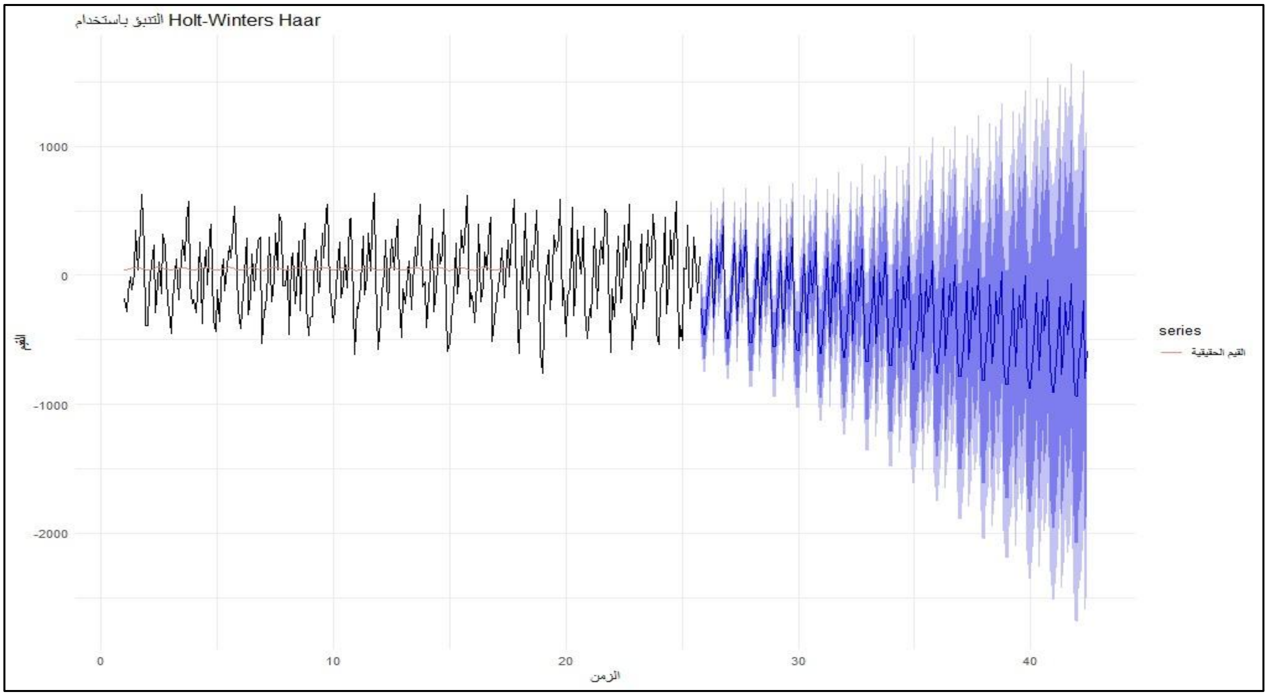

Figure (5): Shows the prediction of the hybrid model using the Haar filter using exponential smoothing and optimal parameter values.

This prompts us to note that the Haar wavelet analysis was used to filter the time series of noise and separate the time series components that represent the general trend and seasonality. This makes hybrid models very useful when the data contains seasonal characteristics and nonlinear fluctuations.

Using the Daubechies filter, the optimal values for the three parameters were obtained, as shown in the table below:

Table (6): Holt-Winters prediction method with the adopted parameter values using the Daubechies filter

Filters	**Optimal parameter values**			**MSD**
	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\boldsymbol{\gamma}$	
Daub	$\alpha = 0.750$	$\beta = 0.006$	$\gamma = 0.64$	25.50

We note from the table that the value of the parameter α = 0.750, indicating that the data still contains some extreme variations after filtering. Similarly, the value of the parameter β = 0.006 indicates that the trend effect is somewhat weak. The value of the parameter γ = 0.64 means that the seasonal effect was high compared to the filter. In general, the Daubechies filter is considered less efficient than the Haar filter because it still retains seasonal variations. As shown in the following hybrid model's prediction:

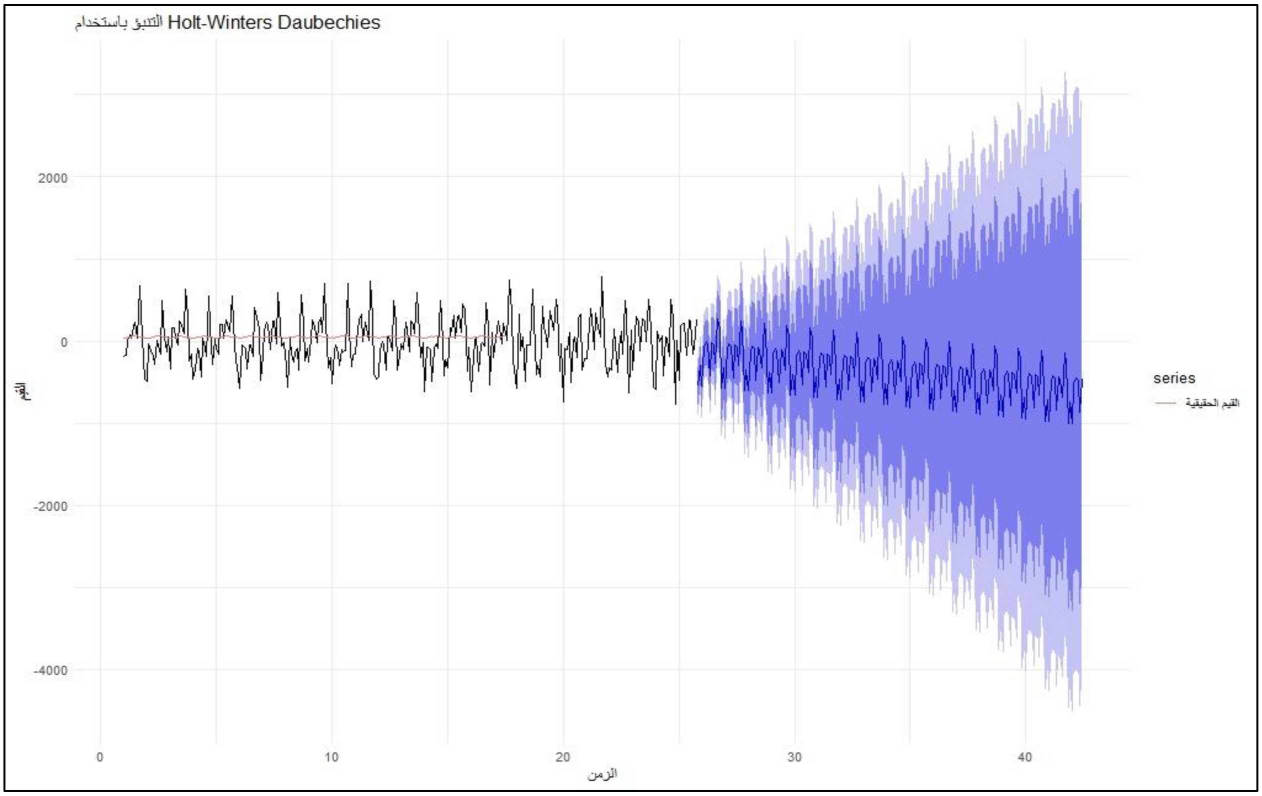

Figure (6): The prediction of the hybrid model using the Daubechies filter using exponential smoothing and optimal parameter values.

The two filters, or what are known as the Haar and Daubechies wavelets, reflect the increase in error over time. However, when using the Daubechies filter, we obtained smoother results that are more appropriate for data with gradual variations.

When using the Coifelet filter, the optimal values for the three parameters were obtained, as shown in the table:

Table (7): Prediction using the Holt-Winters method with the values of the approved parameters using the Coifelet filter

Filters	**Optimal parameter values**			**MSD**
	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\boldsymbol{\gamma}$	
Coif	$\alpha = 0.820$	$\beta = 0.004$	$\gamma = 1.00$	23.32

We note that the value of the parameter $\alpha = 0.820$ indicates that the hybrid model relies on recent values. Also, the value of the parameter $\beta = 0.004$ indicates that the effect of the trend is somewhat weak. Also, the value of the parameter $\gamma = 1.00$ means that there is no clear prediction error through the value of the parameter, meaning that this filter gave a high prediction accuracy compared to the rest of the filters, as shown in the predictive figure of the hybrid model below:

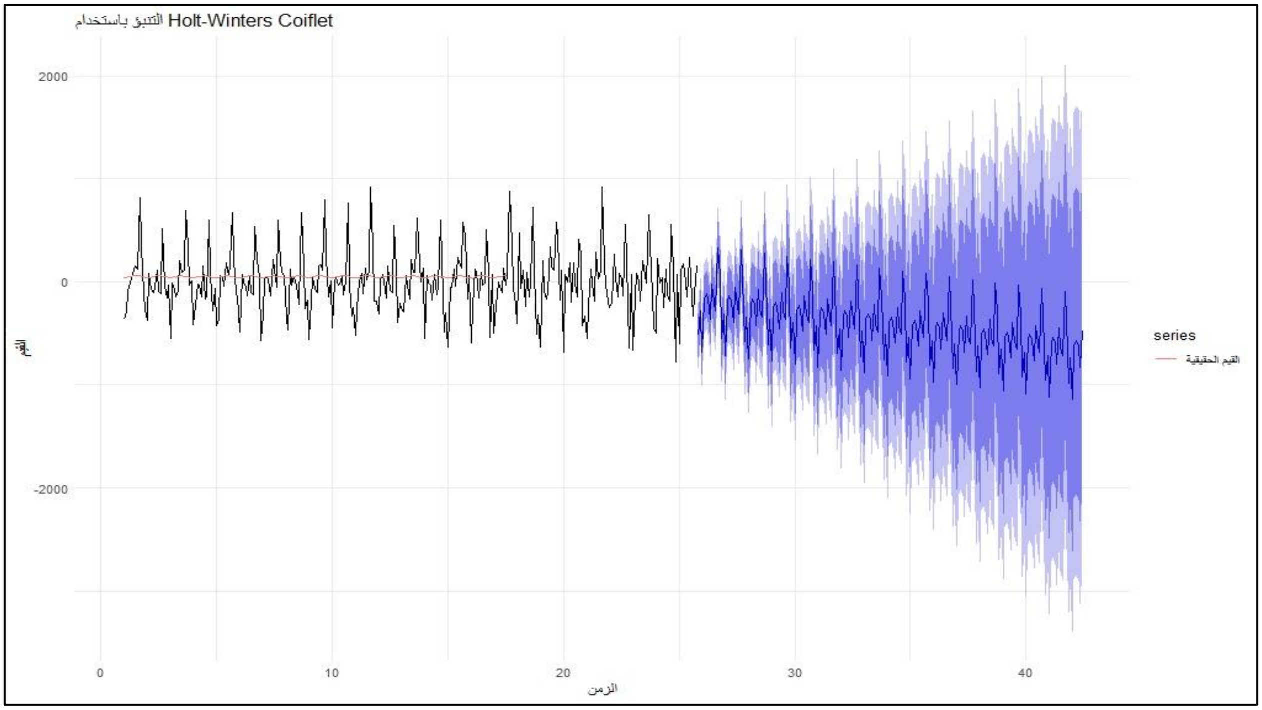

Figure (7): Shows the prediction of the hybrid model using the Coifelet filter using exponential smoothing and optimal parameter values.

Through the analysis using the three wavelets with exponential smoothing using Holt-Winters, it was found that the wavelet that yielded the best predictive values was the Coifelet wavelet. This can be observed through the prediction accuracy test criteria for the three filters, as the Coifelet wavelet corresponds to the lowest values for the prediction accuracy test criteria, as shown in the following table:

Table (8): Prediction accuracy test criteria

	Haar	**Daubechies**	**Coiflet**
MAE	536.2	465.03	320.45
MAPE	53.62	46.50	32.04
RMSE	5.693	5.05	4.82

From the table, we note that the Coifelet wavelet performed best across all metrics, with the lowest MAE = 320.45, lowest MAPE = 32.04, and lowest RMSE = 4.82. Therefore, the Coifelet wavelet can be used in time series analysis, which leads to the best performance in prediction or noise removal.

9. Conclusions:

It was found that increasing the sample size led to an improvement in the accuracy of the obtained models, as well as the accuracy of parameter estimation, as measured by the Mean Standard Deviation of Prediction Errors criterion.

It was found that the Coiflet wavelet filter is the best reliable filter, as it produces the lowest values for the selected samples' MSD criterion, which is a measure of the method's accuracy in noise removal or estimation, for all sample sizes.

10. Recommendations:

Use modern techniques for the same study, including the fuzzy method and neural networks.

Use other wavelets and different methods, and compare the results with this study.

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