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On Semi Strong and Semi Weak Sets in Topological Spaces

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Abstract:

We invest.igate the conc.ept of semi-.strong (s-s.trong) and semi.-weak (s-.weak) sets in topolo.gical spa.ces. We beg.in by giv.ing defini.tions, some exam.ples and fundam.ental characte.ristics for the two sor.ts of sets. The two sets are contr.asted, and the.ir lin.ks are descr.ibed and stud.ied. In addi.tion, a cla.ss of funct.ions usi.ng semi str.ong and semi weak sets has been discov.ered in topological spa.ces. Additi.onally, s-st.rong and s-w.eak sets are indepe.ndent and can.not ever be compa.rable.

Keywords: s-mini.mal semi-open sets, maximal semi-clo. sed sets, s-str.ong sets, and s-weak sets.

1. Introduction

The purp.ose of this stu.dy is to have a more thorough underst.anding of topolo.gical ide.as. The main objec.tive of this sets rese.arch is to supp.ort the advo.cacy quali.ties of the theo.ries. In [1], N. Lev.en (1963) defi.nes semi.-open (s-.open) and semi-closed (s-closed) sets and exam.ines the.ir characte.ristics, in topolo.gical spa.ce, he defi.ned a set A cal.led the (s-.open) set if find an open set $O : O \subseteq A \subseteq \overline{O}$ whe.re $\overline{O}$ deno.ted by the closure of O in Ӽ, the compl.ement semi.-open (s-open) set nam.ed semi-.closed (s-cl.osed) set. In (1971) Semi-c.losure was intro.duced by S. G. Cros.siey and S. K. Hilde.brand in (1971), they defi.ned it as the smallest semi-.closed (s-c.losed) set conta.ined in a set A in topolo.gical spa.ce [2], and short.ened by $Scl(\mathrm{A})$ or $\overline{A}^{s}$. The tru.th $\overline{A}^{s}$ is the inters.ection of all semi clo.sed sets conta.ined A, $\overline{\mathrm{A}}^{s} \subseteq \overline{\mathrm{A}}$ and $\overline{\overline{\mathrm{A}}^{s}}^{s} = \overline{\mathrm{A}}^{s}$. S-mini.mal open and s-ma.ximal clo.sed sets, two significant for.ms of sets that have been expl.ored, were prese.nted in 2003 and 2001, respec.tively. These sets are subcl.asses of the sets s-o.pen and s-cl.osed. Lat.er stud.ies in [5]–[7] loo.ked at s- mini.mum open and s- maxi.mal clo.sed sets as well. This work's goal is to pres.ent cert.ain defini.tions utili.zing s-mi.nimal open and s-ma.ximal clo.sed sets, whi.ch we ref.er to as s-weak sets and s-st.rong sets, respec.tively. We take a look at seve.ral essential characte.ristics and lay some theore.tical groun.dwork for them. Additi.onally, we off.er thr.ee new funct.ions: s-st.rong maxi.mum conti.nuous, s-st.rong maxi.mal conti.nuous, and s-st.rong irres.olute funct.ions. We also exam.ine the rel.ationsh.ips betw.een the.se funct.ions. In [8] R. A. Al- Abdu.lla (2021) defi.nes str.ong and weak sets and expl.ores the.ir characte.ristics. In this work, we pres.ent and stu.dy new clas.ses of s-con.nected spa.ces utili.zing s-st.rong and s-w.eak sets.

Notation:

We sha.ll ref.er to the topolo.gical spa.ce by its sym.bol, (tp-s).

2. Preliminaries

Definition(2. 1): **[9]**

Ass.ume that Ӽ be a (tp-s) and $A \subseteq$ Ӽ. A is cal.led semi.-open (s-open) set in Ӽ iff $A \subseteq \overline{A^{\circ}}$. The compl.ement of s-open set is nam.ed semi-.closed(s-c.losed) that is A s-$clo.sed$ is set iff $\overline{A}^{\circ} \subseteq A$. The inters.ection of all s-cl.osed subs.ets of Ӽ conta.ining A is nam.ed semi-clos.ure (s-closure) of A and the uni.on of all s-open subs.ets of Ӽ conta.ined in A is cal.led semi-inte.rior (s-in.terior) of A, and are deno.ted by $\overline{A}^{s}$, $A^{\circ s}$ respec.tively.

Definition (2.2)[3]: Let (ε,t) be a tripod-s. Then:

(1) If an open set H is in H and diff.ers from the emp.ty set, then H is said to be a minimal open set (or just an m-o.pen set) if all oth.er open sets that sho.uld be incorporated into H are eit.her H or the emp.ty set..

(2) If every clo.sed set that cont.ains F is also a sub.set of F, then the clo.sed set F of F,G is said to be sim.ple or maxim.ally clo.sed (M-c.losed).

Definition(2.3)[2]: If (Ӽ,t) is a tp-s.

(1) If an s-o.pen set H is pres.ent and different from the emp.ty set, then it is cal.led a mini.mal s-o.pen set (or sim.ply an ms-.open set). If eve.ry s-o.pen set that is conta.ined in H is also H or the emp.ty set, then H is said to be an ms-.open set.

(2) If an s-cl.osed set F is in ε, then it is also cal.led a maxi.mum s-cl.osed set (or sim.ply an ms-c.losed set) if the set of open intervals conta.ining F is eit.her the same as F or smal.ler in size.

Definition(2.4)[8]: Let (ε, t) be a tp-s set. If A∘ is an m-o.pen set, then A ≠ ∅: A ⊆ε is cal.led a weak set (or sim.ply a w-set).

If ¯A is an m-cl.osed set, then A is cal.led a str.ong set (or sim.ply a s-set).

3. The Main Findings

Definition(3.1): Let (ε, t) be a tp-s set. There.fore, a prop.erly none.mpty set of A⊆ε is:

1. If A^(µs) is an ms-.open set, it is cal.led a semi.-weak (sw) set.

2. If µ(A)^s is an ms-c.losed set, it is cal.led a semi-.strong (ss) set..

Example(3.2): Cons.ider $X = \{a, b, c\}$ with topo.logy $t = \{\emptyset, X, \{a\}\}$. Then:

Open sets: $\{\emptyset, X, \{a\}\}$

s-o.pen: $\{\emptyset , \{a\}, \{a, b\}, \{a, c\}, X\}$.

Maxi.mal open : $\{a\}$

Maxi.mal s-open : $\{a, b\}, \{a, c\}, X\}$

w-set: $\{ X, \{a\}, \{a, b\}, \{a, c\}\}$.

sw-set: $\{ X,\{a, b\}, \{a, c\}\}$.

Remark (3-3): Let (Ӽ , t) be a tp-s then:

1- Not all open set is maxi.mal open as in **[**Exam.ple(3.2)**]**show that $\{a\}$ is open but is not maximal open set**.**

2- Not eve.ry maxi.mal s-o.pen set is open as in **[**Exam.ple(3.2)**]**show that $\{a, b\}$ is maxi.mal s-open but is not open set**.**

3- Not eve.ry open set is Weak set as in **[**Exam.ple(3.2)**]**show that $\{\emptyset\}$ is open but is not Weak set**.**

4- Not eve.ry Weak set is open set as in **[**Exam.ple(3.2)**]**show that $\{a, b\}, \{a, c\}$, is Weak set but is not open set**.**

5- Not eve.ry s-open set is sw-set as in **[**Exam.ple(3.2)]show that $\{a\}$ is s-open but is not sw-set**.**

6- Not eve.ry open set is sw-set as in the [Exam.ple(3.2)]sho.ws.

that {a} open sets but is not sw-set beca.use $\{a\}^{\circ s} = \{a\}$, whi.ch is not maxi.mal semi.-open in Ꭓ.

Proposition(3.4): Let Ꭓ be tp-s then the union of all sw-sets in Ꭓ is also sw-set.

Pro.of:(cl.ear)

Proposition(3.5): Let ε be a set oftps. Next, the meet.ing of two sets in ε that are both smal.ler than or equ.al to a thi.rd set is not a set in ε, this is demons.trated in the exam.ple (3.2). Obvio.usly, {a,b} and {a,c} are both subs.ets of ε, but {a,b} ∩ {a,c} = {a} is not a sub.set..

Theorem(3.6): All.ow (ε,t) to be the tp-s set, and H to be the compl.ement of it. Then H is an Ms-c.losed set if and only if H^c is an Ms-.open set.

Proof:

Let A be a maxi.mal s-cl.osed set, A⊂Ꭓ or A⊂A, there.fore 〖∅⊂A〗^c, hen.ce A⊂Ꭓ or A⊂A, whi.ch mea.ns A is a maxi.mal s-o.pen set..

Theorem(3.7): Let (ε,t) be a tp-s.uple, and A be an s-o.pen set in ε. Then A is consi.dered an open set and only if it is consi.dered an open set..

Proof:

Let A be a set that is both s- and an w-cl.osed set. Then A^(°s) is an open set that is ms-bo.unded, and A = A^(°s). As a res.ult, A is an ms-co.mplete set. Alternatively, cons.ider the case whe.re A is an ms-co.mplete set. Then A is an open set, so A^(0) is also an open set. As a res.ult, A is a set of swa.ps..

Example(3.8): All.ow (R, t_u) to be a tp-system, whe.re R is the dom.ain of real numb.ers and t_u is the stan.dard topo.logy. (R - 1) is there.fore an s-o.pen set, alth.ough it is not an ms-.open or a sw-.open set. Desp.ite 1's lack of ms-.ness or ss-.ness, it sti.ll consti.tutes an s-cl.osed set..

Theorem(3.9): Let (ε, t) be a tp-sys.tematic set that incl.udes the inters.ection of any two open sets in ε as a part of its sys.tem. Next, an s-o.pen sub.set of ε is consi.dered an ss set if and only if its compl.ement, H^c, is an sw set..

Proof:

Let $\mathcal{H}$ be ss-set and $\mathcal{U}$ an s-open set like that $\mathcal{U} \subseteq \mathcal{H}^{c^{\circ s}}$. Then $\overline{\mathcal{H}}^{s} \subseteq \mathcal{U}^{c}$. Sin.ce $\mathcal{U}$ is ss-set and inters.ection of two s-open set in Ꭓ is s-open set, then $\overline{\mathcal{H}}^{s}$ is Ms-cl.osed. There.fore $\mathcal{U}^{c} = Ꭓ$ or $\mathcal{U}^{c} = \overline{\mathcal{H}}^{s}$. Hen.ce $\mathcal{U} = \emptyset$ or $\mathcal{U} = \mathcal{H}^{c^{\circ s}}$. There.fore $\mathcal{H}^{c^{\circ s}}$ is ms-open set. Then $\mathcal{H}^{c}$ is sw-set. Conve.rsely, let $\mathcal{H}^{c}$ be sw-set and $\mathcal{F}$ be a clo.sed set with $\overline{\mathcal{H}}^{s} \subseteq \mathcal{F}$. Then $\mathcal{F}^{c} \subseteq \mathcal{H}^{c^{\circ s}}$. Sin.ce $\mathcal{H}^{c}$ is

sw-set, then $\mathcal{H}^{c^{\circ s}}$ is ms-open. There.fore $\mathcal{F}^{c} = \emptyset$ or $\mathcal{F}^{c} = \mathcal{H}^{c^{\circ s}}$. Hen.ce $\mathcal{F} = X$ or $\mathcal{F} = \overline{\mathcal{H}}^{s}$. There.fore $\overline{\mathcal{H}}^{s}$ is Ms-clo.sed set. Then $\mathcal{H}$ is ss-set.

Example (3.10): Supp.ose (R,t) is a tp-s set, whe.re R is the set of all real numb.ers, with the topo.logy; t={A∋R:1∈A}∪{∅}. Then {1} is an open set and a set of wor.ds, but it isn't a clo.sed set or an ss set. R-{1} is also consi.dered an ms-c.losed set and an ss-.set, but not an ms-.open set or a swim.ming set.

Theorem(3.11): Let (X, t)be a tp-s and $A \subseteq X$. Then if A is sw-set, $B \subseteq X$ such that $B^{\circ s} \neq \emptyset$ and $B \subseteq A$ then $A^{\circ s} = B^{\circ s}$.

Proof:

Let A be sw-set, $B \subseteq X$ such that $B^{\circ s} \neq \emptyset$ and $A \subseteq B$. Then $A^{\circ s}$ is ms-open set and $B^{\circ s} \subseteq A^{\circ s}$. There.fore $A^{\circ s} = B^{\circ s}$.

Theorem (3.12): If (ε, t) is a tp-.slot. If A is an ss set, F is an s-cl.osed set, and if ¯(A)^s∪F≠ε, then F⊆ ¯(A)^s.

Proof:

Given that A is ss-set, then $\overline{A}^{s}$ is Ms-clo.sed set, also due to$\left(\overline{A}^{s} \cup \mathcal{F}\right)$is s-clo.sed set, $\overline{A}^{s} \subseteq \left(\overline{A}^{s} \cup \mathcal{F}\right)$ then $\left(\overline{A}^{s} \cup \mathcal{F}\right) = \overline{A}^{s}$. Hen.ce. $\mathcal{F} \subseteq \overline{A}^{s}$.

Corollary (3.13): Supp.ose (ε, t) is a tp-.set that cont.ains the inters.ection of any two open sets in ε as a sub.set. If A is a set of swa.ps, G is an open set that cont.ains A^(εs) and is there.fore incl.uded in A. If A is a set of swa.ps that incl.udes the emp.ty set as a compo.nent, then A is said to be emp.ty..

Proof:

Sin.ce A is an s-set, A(°s) is an Ms-.door set. Since (A^.(°s)+G) is an s-o.pen set, A^(.°s)+G is also an s-o.pen set. Sin.ce A(°s) ∩ G is not emp.ty, there.fore A(°s) ⊇ G..

Corollary(3.14): If $\mathcal{F}, \mathcal{B}$ are ss-sets in a tp-s (X, t)such that $\overline{\mathcal{F}}^{s} \cup \overline{\mathcal{B}}^{s} \neq X$ then $\overline{\mathcal{F}}^{s} = \overline{\mathcal{B}}^{s}$.

Proof:

Let $\mathcal{F}$,$\mathcal{B}$ be ss-sets such that $\overline{\mathcal{F}}^{s} \cup \overline{\mathcal{B}}^{s} \neq X$. Then $\overline{\mathcal{B}}^{s}$ is s-clo.sed set. There.fore, $\overline{\mathcal{B}}^{s} \subseteq \overline{\mathcal{F}}^{s}$. By the same way we can pro.of $\overline{\mathcal{F}}^{s} \subseteq \overline{\mathcal{B}}^{s}$. Then $\overline{\mathcal{F}}^{s} = \overline{\mathcal{B}}^{s}$.

Corollary (3.15): If $\mathcal{U}, \mathcal{V}$ are w-sets in a tp-s (X, t) with $\mathcal{U}^{\circ s} \cap \mathcal{V}^{\circ s} \neq \emptyset$, then $\mathcal{U}^{\circ s} = \mathcal{V}^{\circ s}$.

Proof:

Let $\mathcal{U}, \mathcal{V}$ be w-sets such that $\mathcal{U}^{\circ s} \cap \mathcal{V}^{\circ s} \neq \emptyset$. Then $\mathcal{V}^{\circ s}$ is s-open set. There.fore, $\mathcal{U}^{\circ s} \subseteq \mathcal{V}^{\circ s}$. By the same way we can pro.of $\mathcal{V}^{\circ s} \subseteq \mathcal{U}^{\circ s}$. Then $\mathcal{U}^{\circ s} = \mathcal{V}^{\circ s}$.

Theorem (3.16): Let (μ,t) be the tp-.slot, A be the set of slo.ts, and x be in A^(0°). Next, if s is an open set with s-o.pen sets W, if x is not in W, then A^(°s) is a sub.set of W^c.

Proof:

Suppo.sons que x parti.cipe à la rota.tion aux pôl.es. Alo.rs, pour tout ense.mble ouv.ert s W, si x n'appa.rtient pas à W, alors A^(°s) ⊈ W. Par conséque.nt, A^(°s) est inclus dans W^c.

Corol.lary (3.17): Let (ε, t) be a tp-s.uple, and A be an ssu.ple, such that x / ∈ ¯(A)^s. Next, if s is a clo.sed set that cont.ains x, then F^c is incl.uded in the set of all poss.ible open.ings of the form A^s.

Proof:

Let A be a set of ss wor.ds, and x ∉ A^s. Next, if x is in a s-cl.osed set F, then F ⊈ ¯(A)^s. As a res.ult, F^c is incl.uded in the inte.rior of A's sph.ere of influ.ence. Theo.rems (3.18): Let (ε, t) be a tp-.slot. If ε has two subs.ets A and B that are conta.ined in t, and the inters.ection of two open sets in ε is sti.ll an open set, then A^(°s) ⊈ B. Additi.onally, if A and B have the prop.erty that A^(°s) ∩ B = ∅, then A and B are said to be adja.cent in t.

Pro.of: Let A be a set of car.ds with a specific distri.bution. Then A^(°s) is an open set that is sm-complete. Sin.ce A^(°s) ∩ B is conta.ined in A^(°s) and A^(°s) ⊈ B, there.fore A^(°s) ∩ B = ∅.

Corol.lary (3.19): Let (ε, t) be a tp-s.ystem, such that the inters.ection of two s-o.pen sets in ε is sti.ll an s-o.pen set. If ε has a set A with an s-closed compo.nent B, and B ⊈ ¯(A)^s, then ¯(A)^s ∪ B = ε.

Proof:

Theo.rems (3.20): Let (ε, t) be a tp-s set, and A and B be s-o.pen sets in ε. If A is a set of swa.ps, and B is not emp.ty and B is incl.uded in A, then B is a set of swa.ps.

Pro.of: Let A be a set of size s, and B be a non-.empty open set of size t, and B ⊆ A. Next, A^(°s) is an open set that is ms-co.mplete. If G is an s-o.pen set, then G is incl.uded in B^(°s). Then G is incl.uded in A^(°s), there.fore G is eit.her emp.ty or A^(°s) is equ.al to G. If A^(°s) is equ.al to G, then G is said to be B^(°s). This impl.ies that B is a ser.ies of orde.red sets (so).

Theo.rems (3.21): Let (ε, t) be a set of sor.ted collec.tions (tp-s), and A and B be s-o.pen sets in ε. If A is a set of collec.tions of obje.cts that are orde.red by some prop.erty, and the.re is no elem.ent in A that is smaller than B, then B is a set of orde.red collec.tions (so).

Pro.of: Let A be a set of sor.ted collec.tions (sw), and B be a non-.empty s-o.pen set that is not in A. Next, A^(°s) is a colle.ction of sets that are orde.red by the.ir memb.ers' nam.es (ms-.close). Let G be an open spa.ce, and G ⊆ B^(0°). Then G is incl.uded in A^(°s), there.fore G is eit.her emp.ty or A^(°s) is equ.al to G. If A^(°s) is equ.al to G, then G is said to be B^(°s). This implies that B is a ser.ies of orde.red sets (so).

Note (3.22): If (ε, t) is a tp-s.ystem, and A is a sub.set of it. If A is an ss set, then the set of its elements, ¯(A)^s, is also an ss set. If A is a set of swa.ps, then A^(°s) is also a set of swa.ps.

Corol.lary (3.23): Let (ε, t) be a tp-s.ystem, and A, B be par.ts of it. If A is an ss set, and A ∪ B ≠ ε, then A ∪ B is an ss set.

Corol.lary (3.24): Let (ε, t) be a tp-s.ystem, and A, B be compo.nents of t. If A is a set of swa.ps, and if A ∩ B is not empty, then A ∩ B is also a set of swa.ps. Theo.rem (3.25): Let H be a non-.empty sub.set of the tp set (ε, t) that satis.fies the follo.wing condi.tions: the inters.ection of any two s-open sets in ε is also an s-o.pen set. As a res.ult, the follo.wing state.ments are synon.ymous.

If H is an s-o.pen set, then it is an s-w set.

If H is an s-cl.osed set, then it is an ss-.set.

Pro.of: Theo.rems (3-9) and (3-14) are suppo.rtive of this pro.of.

Theo.rem (3.26): Let H be a proper sub.set of tps(ε, t) that inter.sects any two s-open sets in ε is also an s-o.pen set. As a res.ult, the follo.wing state.ments are synerg.istic:

If H is an ms-.open set, then it is also an sw-.set.

If H is a ms-co.mplete set, then it is an ss-.set.

Pro.of:

The theo.rems (3-6) and (3-9) endo.rse this pro.of.

Theo.rem (3.27): Let $\mathcal{U}$ be a sub.set of tp-s (ε, t) that incl.udes the inters.ection of any two s-o.pen sets in ε as well. After that, the follo.wing state.ments are synon.ymous:

If $\mathcal{U}$ is a set of car.ds with an ace as its fir.st elem.ent, then $\mathcal{U}$ is an open set with a ms-.root.

If $\mathcal{U}$ is an ss-.set, then it is an ms-b.ound set.

Pro.of:

The theo.rems (3-6) and (3-9) endo.rse this pro.of.

Theo.rem (3.28): Let (ε, t_y) be a subs.pace of tp-s (ε, t) that is open and clo.sed. If A is an s-cl.osed set in ε, and ε ⊈ ¯(A)^s and A ∩ ε is not emp.ty, then A ∩ ε is also an s-cl.osed set in ε.

Pro.of:

Let F be an s-cl.osed set in ε, and y (or c) be a poi.nt in F. Then F is an s-cl.osed set in ε, and A is a sub.set of F conta.ining y (or c). As a res.ult, A is also an s-cl.osed set in c. Sin.ce A is an s-set, its compl.ement, A^c or F, is also an s-set. If F ∪ ε^c = ε, then F = ε. If the set of param.eters assoc.iated with A is incl.uded in the set of parameters associated with F and if s y is a new param.eter assoc.iated with A and y is a new param.eter assoc.iated with F and s y . As a res.ult, y is incl.uded in the set A∩y.)^(s y) = F. As a res.ult, A∩Ƴ is an ss-.set in Ƴ.

Theo.rems (3.29): Let (Ƴ, t_y) be a clo.sed subs.pace of tp-s (Ϫ,t).

If A is an ss-.set in ε, then eit.her ¯(A)^s or ¯(A)^s ∪ ε = ε.

Pro.of:

The pro.of is deri.ved from the set of rul.es ¯(A)^s ⊆ ¯(A)^s ∪ ε.

Theo.rems (3.30): Let (ε, t_y) be a subs.pace of tp-s (ε, t). If A is a set in the enviro.nment of size 3, and if A is also a set in the enviro.nment and if A and ε inter.sect, then A is also a set in the enviro.nment.

Pro.of:

Assu.ming that G is a sub.set of t_y that satis.fies the following condi.tions: G is a sub.set of 〚(A ∩ ε) 〛 ^(° ε). Then G is the squ.are root of ε, and G = G ∩ ε ⊆ (A ∩ ε)^(°s ε) ∩ ε^(∘s) = (A ∩ ε)^(∘s) = A^(∘s) ∩ ε^(∘s) ⊆ A^(∘s). As a res.ult, G is equ.al to A^(∘s) or G is emp.ty. If G = A^(∘s), then (A ∩ ε)^(°s ε) = A^(°s ε) ∩ ε^(°s ε) = A^(°s ε) ∩ ε^(∘s) = A^(∘s) = G. As a res.ult, A ∩ ε is the square root of ε.

Theo.rem (3.31): Let (Ƴ, t_y) be a subs.pace of tp-s (Ϫ,t) that is open. If A is a set of ang.les in degrees, then A^(°s)⊆Ƴ or A^(°s)�Ƴ =∅.

Pro.of:

The concl.usion is derived from A^(.°s)∩Ƴ⊆A^(°s).

Theo.rem (3.32): Let (Ƴ, t_y) be a clo.sed subs.pace of tp-s (Ϫ,t). If eve.ry none.mpty, pro.per, and ope.n-s sub.set of Ϫ is a set of sw in Ϫ, then eve.ry none.mpty, pro.per, and open-s sub.set of Ƴ is also a set of sw in Ƴ.

Pro.of: Let U' be an ope.n-s sub.set of B. Then U is an open s- sub.set of ε. As a res.ult, U is a set in ε. Sin.ce U ∩ ε is diffe.rent from ε and U ∩ ε is diffe.rent from noth.ing, by Theo.rem (3-32), U is a sub.set of ε.

Theo.rems (3.33): Let (ε, t_y) be a subs.pace that is both open and clo.sed in tp-s (ε, t). If eve.ry non-.empty pro.per sub.set of ε that is not emp.ty is an sset of ε, then every non-empty proper sub.set of ε that is not emp.ty is an sset in ε.

Pro.of:

Supp.ose F is a non-.empty s-cl.osed sub.set of ε. Then F is a s-com.plete sub.set of ε. As a res.ult, F is a sub.set of ε. Sin.ce F∩Ƴ.≠Ƴ and F∩Y≠∅, accor.ding to Theo.rem (3-31), F is a set that is ss in ε.

4- Some applications employ semi-intense and semi-fragile sets:

Defin.ition (4.1): Let ƒ: ε ⟶ε be a transfo.rmation from the fir.st to the sec.ond type ofε. Aft.er that, the proc.ess is refe.rred to as follows:

SS-st.rongly maxim.ized conti.nuous (ssM-co.ntinuous for sho.rt), if eve.ry ss-.set A in ε, ƒ^(-1)(A) is an Ms-c.losed set in ε.

Maxi.mal ss-st.rongly conti.nuous (mss-co.ntinuous for sho.rt), if eve.ry ss-c.losed set A in ε, ƒ^(-1)(A) is an ss-.set in ε.

SS-.very indecipherable (ss-in.effable for sho.rt), if eve.ry ss-.set A in ε, ƒ^(-1)(A) is an ss-.set in ε.

SS-very indecipherable (ss-in.effable for sho.rt), if eve.ry ss-.set A in ε, ƒ^(-1)(A) is an ss-.set in ε.

Theo.rem (4.2): Let ƒ:ε ⟶ε be a mapp.ing from a tp-s on ε to a tp-s on ε. Next, the follo.wing two state.ments are equa.lly val.id:

The stock's perfor.mance is consi.dered good.

Eve.ry sub.set of A in ε that is conta.ined in the set A is an ms-.open set on ε.

Theo.rem (4.3): Let ƒ:ε ⟶ε be a mapping from a trip.od's poi.nt of view on ε to a trip.od's poi.nt of view on ε. Next, the follo.wing two state.ments are equa.lly val.id:

ƒ is Mss-con.tinuous.

Every ms-.open set A in ε, whi.ch is conta.ined in an open set of size at most ms, is also an ms-.open set on ε.

Theo.rem (4.4): Let ƒ:ε ⟶ε be a mapp.ing from a tp-s on ε to a tp-s on ε. Next, the follo.wing two state.ments are ident.ical:

The mean.ing of ss is ambig.uous.

Eve.ry subsy.stem of A in ε is also a subsy.stem of A in ε.

Theo.rem (4.5): Let ƒ:ε ⟶ε be a mapp.ing from a tp-s set in ε to a tp-s set in ε.T.hen:

If ƒ is conti.nuous, then it is undec.ided.

If ƒ is undec.ided, then it is Mss-pe.riodic.

As a res.ult, if f is ssM-pe.riodic, then it is Mss-peri.odic.Proof:

The pro.of is comp.lete, sin.ce eve.ry sM-co.vered set is an ss set.

Exam.ple: (4.6) Let the set of param.eters be ε = {k, v, l}, whe.re t = {{k}, ∅, ε}, and ε' = {k, v}, whe.re t' = {{k}, ∅, ε}. Then {l} is an ss-.set in ε, but not a ssM-co.mplete set. Let ƒ:(ε,t)⟶(ε,t') be a func.tion that satis.fies the follo.wing prope.rties: if t is grea.ter than or equ.al to k, then ƒ(k) = t, and if t is smal.ler than k, then ƒ(k) = k. Then, ƒ is ss-indet.erminate, but not ssM-con.tinuous, beca.use for the ss set {v} in Ƴ, the func.tion ƒ^(-1) {v} is equ.al to k, and k is not an sM-c.losed set in ε.

Theo.rem (4.7): Let ƒ:ε → ε be a map from the tp-s set ε to the tp-s set Ƴ, whe.re ε is defi.ned as foll.ows: eve.ry ss set is an sM-c.losed set. If ƒ is una.ble to be reso.lved, then ƒ is consi.dered to be ssM-con.stinuous.

Pro.of:

The concl.usion is deri.ved from the prem.ises. Sam.ple (4.8): Let ε = {k,v,l}, whe.re t = {{k,v},∅, ε}, ϒ = {k,v,l,h}. Let t' = {k,v}, ∅, andϒ. Then {l} is an open set in ϒ, but not a clo.sed set with resp.ect to sm. Let ʄ:(Ҳ,t)⟶(ϒ,t') be a function that satis.fies the follo.wing prope.rties: ʄ(k) = k, ʄ(v) = v, and ʄ(l) = l. Then ʄ is mss-con.tinuous, but not ss-indet.erminate, beca.use for the ss set {h} inϒ, ʄ^(-1)(h) =∅ is pres.ent, whi.le ʄ^(-1)(h) =∅ is not an sm-c.losed set in Ҳ.

Theo.rem (4.9): Let ʄ: ε⟶ε be a func.tion from the tp-s.phere to the tp-s.phere, whe.re the prop.erty that eve.ry ss set is sM-c.losed is satis.fied by ε. If the let.ter ʄ is Mss.con.tiguous, then it is ss-ind.istinct.

Pro.of: The pro.of is deri.ved from the prem.ises dire.ctly.

Exam.ple (4.10): In [Exa.mple (4-8)], ʄ is conti.nual. Sin.ce ʄ is not indeter.minate in this exam.ple, and accor.ding to the theo.rem (4-5), every ssM-con.tinuous func.tion is also indeter.minate, there.fore, ʄ is not ssM-con.tinuous.

Theo.rem (4.11): Let ʄ: ε ⟶ε be a func.tion from the tp-s.phere of ε to the tp-s.phere of ε, and let ε, ε be an sM-c.losed ss-.set. If ʄ is conti.nual, so is also ssM.

Pro.of:

The pro.of is deri.ved from the prem.ises dire.ctly.

Theo.rem (4.12): Let (ε, t) and (ε, t') be two tp-s, and ʄ: (ε, t) ⟶(ε, t') be a surje.ctive, clo.sed, and conti.nuous func.tion. If A is an ss-.set in ε, then so is also ʄ(A)..

Proof:

All.ow A to be an elem.ent of the set ε. Next, A^s is an Ms-co.mplete set in ε. Let F be a clo.sed set in ε that cont.ains the ori.gin of the rota.tion, and let A be a set of the form [0, 1] × [0, 1]. Then A is incl.uded in F, and thus A is also incl.uded in the set of poi.nts of the form [0, 1] × [0, 1]. Sin.ce ʄ is a continual func.tion, the set of all poss.ible val.ues for ʄ is incl.uded in the set of all poss.ible val.ues for ƅ(-1)(F). Sin.ce A is an s-set, the set of indi.ces of the form (1, 0, 1) or (0, 1, 0) is emp.ty or the set of indi.ces of the form (1, 0, 1) or (0, 1, 0). If (-1)(F) = {} and if ʄ is a surje.ctive map, then ʄ = {}. If ʄ^(-1)(F) = ¯A^s and ʄ is a surje.ctive, conti.nuous, and clo.sed func.tion, then F = ¯(ʄ.(A))^s. As a res.ult, the set of Ms's is clo.sed. As a res.ult, ʄ(A) is an ss-.set.

Corol.lary (4.13): Let (ε, t) and (γ, t') be two tp-.sets, and ʄ:(ε, t)⟶(γ, t') be a bijec.tive, clo.sed, and continuous func.tion. If A is a set in the enviro.nment ofε, then ʄ(A) is a set in the enviro.nment ofγ.

Pro.of: Let A be a set of size at lea.st 3 in the enviro.nment of the set ε. Then A^c is an ss-.set in ε. Accor.ding to Theo.rems (4-12), the set of ss in A^c is a set of ss in ϒ. As a res.ult, so is ϒ - ʄ(A^c) a set of ss in the set. Sin.ce ʄ is surje.ctive, so is its range, whi.ch is a set of solut.ions to the sys.tem of equat.ions in the form of a vec.tor in R^i.

Theo.rem (4.14): Let (ɼ, t) and (ϒ, t^') be two tp-s, and ʄ:(ɼ, t) ⟶(ϒ, t^') be a bijec.tive, open, and conti.nuous func.tion. If A is a set of ss in ϒ, then ʄ^(-1)(A) is also a set of ss in ɼ.

Pro.of:

All.ow A to be a colle.ction of ss's in ϒ. Then ¯A^s is a Ms-co.mplete set in ϒ. Let F be a clo.sed set in U that is also s-cl.osed, and let ¯(ʄ^.(-1)(A))^s be a sub.set of F. Then ʄ^(-1)(A) is a sub.set of F, and sin.ce ʄ is surje.ctive, A is a sub.set of ʄ(F). Sin.ce ʄ is an s-cl.osed func.tion, the dom.ain of its act.ion is conta.ined in the set ʄ(F). Sin.ce the set of Ms's is clo.sed in ϒ, ʄ(F) = ∅ or ʄ(F) = ¯A^s. If the set of frie.nds of F is empty, then F is also emp.ty. If ʄ(F) = ¯A^s and ʄ is a surje.ctive, open, and conti.nuous func.tion, then F = ʄ^(.-1)(ʄ(F)) = ʄ^(-.1)(¯A^s) = ¯(ʄ^.(-1)(A))^s. As a res.ult, ʄ^(-1)(A) is an ss set in E.

Corol.lary (4.15): Allo.wing for two diffe.rent tp-s, and a bijec.tive, open, and continuous func.tion between them, we have the follo.wing: (ε, t) and (ε, t') are both s-tp's, and (ε, t) ⟶(ε, t') is a conti.nuous, s-o.pen, and bijective func.tion. If A is a sub.set of the set of numb.ers ε, then the set ƒ^(-1)(A) is also a sub.set of the set of numb.ers ε.

Pro.of:

Assu.ming A is a mem.ber of the set Ƴ. Then A^c is a set in the ss set. As a result, accor.ding to Theo.rem (4-14), ƒ^(.-1)(A^c) is a set of size 1 in the dom.ain of ε. As a res.ult, the set of numb.ers ε - (ƒ^.(-1)(A^c)) is a set of meas.ure zero in the set of numb.ers ε. As a res.ult, ƒ^(-1)(A) is a set in E that is conta.ined in the inte.rior of the cir.cle.

Theo.rem (4.16): Let (ε,t), (ε,t'), and (μ,t'') all be tp-conti.nuous.Then:

If the func.tion f:(ε,t) → (ε,t') is ssM-c.onstan and the func.tion g:(ε,t) → (μ,t'') is Ms-co.nstan, then the func.tion g:(ƒ:(ε,t) → (μ,t'') is also Ms-co.nstan.

If the func.tion f:(ε,t) → (ε,t') is conti.nuous and the func.tion g:(ε,t) → (μ,t'') is also continuous, then g:(f:(ε,t) → (μ,t'')) is also Ms-con.tinuous. The map that assoc.iates (θ, t) with (θ, t') is sM-cons.tiguous, there.fore g(θ, t') with (θ, t) is ss-inde.termine and Ms-cons.tiguous.

If the slo.pe of (t, If t) is the same as that of (u, If u), then g(t, t') = g(u, t') and sM is conti.nuous. As a res.ult, the trans.ition from g to Ms is indeter.minate and the trans.ition from Ms to g is also indeter.minate.

If (Ƴ,t^') is Ms's conti.nuous and sM's continuous, then g∘ƒ:(θ,t)□(Ȥ,t^'') if ƒ:(θ,t)→(Ȥ,t^'') is s-indet.erminate. As a res.ult, if (θ,t) → (Ȥ,t^'') is ss-indet.erminate, and (Ƴ,t^') → (Ȥ,t^'') is Ms-con.tinuous, then g∘ƒ:(θ,t)→(Ȥ,t^'') is also Ms-con.tinuous.

If (θ,t) → (Ƴ,t^'), and the set g:(Ƴ,t^') → (Ȥ,t^'') is ss-indet.erminate, and g:(Ƴ,t^') → (Ȥ,t^'') is sM-con.tinuous, and if (Ӽ,t) → (Ȥ,t^'') is ss-indet.erminate and... Ms-con.tinuous.

If (Ӽ ,t) → (Ƴ,t^') is ss-indet.erminate and g:(Ƴ,t^') → (Ȥ,t^'') is ss-indet.erminate, then g∘ƒ:(Ӽ ,t) → (Ȥ,t^'') is ss-indet.erminate and Ms-con.tinuous.

Pro.of:

The pro.of is as foll.ows: eve.ry Ms-c.losed set is equiv.alent to an ss-.set, and by Theo.rem... <4-5> Eve.ry Ms-c.losed set can be repres.ented by an ss-set, thus the pro.of is comp.lete.

Exam.ple: (4.17) has the form Let ε = Ȥ = {k, v, l, h} and ε = {k, v, l}, whe.re t = Let {{k, v}, ε, ∅}, t' = {{k, v}, ε, ∅} and t'' = {{k, v}, Ȥ, ∅}. Let ƒ:(ε, t) → (ε, t'), whe.re ƒ(k) = k, ƒ(v) = v, ƒ(l) = ƒ(h) = l, and g:(ε, t') → (ε, t''), whe.re g(k) = k, g(v) = v, g(l) = l. Then ƒ is sM-co.nstan and s-inde.termine. G is cal.led Ms-con.tinuous, and g is also cal.led Ms-con.tinuous, but g∘ƒ is not m-inde.pendent beca.use (h) is an s-set in Ȥ, but (g∘ƒ)^.(-1)(h)=∅ is not an s-set in ε. By the theo.rem's descri.ption, g∘ƒ is also not sM-te.mpered.

Exam.ple (4.18): Let ε = ε = Ȥ = {k,v,l,h}, whe.re t = {{k,v},{k},ε,∅}, t' = {{k.,v},{k,v,l},ε,∅}, t''' = {{k,v,l},Ȥ,∅}. Allo.wing (ε,t) to evo.lve to (ε,t'), and lett.ing g:(ε,t') to evo.lve to (Ȥ ,t''), is what enab.les us to def.ine the iden.tity func.tion. Then ƒ is indecom.posable and Ms.en.dless, and g is also indecom.posable and Ms.en.dless, but g∘ƒ is not Ms.en.dless beca.use {h} is a set of s. It's set in Ƴ, but (g∘ƒ)^(-1) (h)=h isn't in E. Ms-c.losed.

Exam.ple: (4.19) has the form of a list of 4 elem.ents, each of whi.ch is of the form {k,v,l,h} and has a corresp.onding elem.ent of the form {k,v,l}. t has the form {{k,v},{k},ε,∅}, t' has the form {{k.,v},{k,v,l},ε,∅}, and t^'' has the form of a list of 4 elem.ents, each of whi.ch is of the form {k,v,l,h}. Let ƒ:(ε ,t) → (Ƴ,t^'), whe.re ƒ (k)=k, ƒ (v)=v, ƒ (l)=l, ƒ (h)=h, and g:(Ƴ,t^') □ (Ȥ ,t^''), whe.re g (k)=k, g(v)=v, g(l).=g(h)=l. Then ƫ is indecom.posable, Ms..end, g is indecom.posable, but g∘ƫ is not Ms..end beca.use {l} is an s-set in Ⱬ, but 〖(g∘ƫ)〗^(-.1)(l)={l,h} is not M-cl.osed in ε.

Exam.ple: (4.20) has the form t={.{k,v},{k},Ҳ,∅}, t^'=.{{k,v},{k,v,l},Ƴ,∅} and t^'.'={{k,v},Ⱬ,∅}. The defin.ition of g is as foll.ows: g (k)=k, g (v)=g (l)=v, g (h)=l. Then Ms is consi.stent, s is ambig.uous, and G∘ƫ is consi.stent but not consi.stent, beca.use {l} is M's clo.sed dom.ain in Ⱬ, but 〖(g∘ƫ)〗^(-1)(l)= ∅ is not a set in Ҳ. Accor.ding to Theo.rem 4.5, g∘ƫ is not s (ineq.uality) nor sM (end).

Theorem(4.21): Let (ε, t), (ε, t'), and ((μ, t'')) be sets that are tp-s, and let ε have the prop.erty that eve.ry set is M-clos.ed.Then:

If the mat.rix: (ε, t) → (ε, t') is Ms-con.tinuous, and the vec.tor: (ε, t') → (μ, t'') is sM-con.tinuous, then the mat.rix: g: (ε, t) → (μ, t'') is also sM-con.tinuous.

If the matr.ix:(ε,t) → (ε,t'') is indecom.posable, and the vect.or:(ε,t) → (ε,t'') is continuous, then the mat.rix: g:(ε,t) → (ε,t'') is also conti.nuous.

If f: ƫ:(ε,t) → (ε,t'') is s-indet.erminate, and g:(ε,t) → (ε,t'') is also s-indet.erminate, then g: ƫ:(ε,t) → (ε,t'') is also sM-con.tinuous.

Theo.rem (4.22): Let (ε,t), (ε,t'), and (μ,t'') be sets that are tp-s, and let ε have the property that eve.ry s set is M-clos.ed.Then:

If (ε,t) → (ε,t') is Ms-con.tinuous, and (g,t) → (μ,t'') is Ms-con.tinuous, then (g∘)ƫ:(ε,t) → (μ,t'') is Ms-con.tinuous.

If (ε,t) → (ε,t') is Ms-con.tinuous, and (g,t) → (μ,t'') is Ms-con.tinuous, then (g,t):(ε,t) → (μ,t'') is Ms-con.tinuous. Sin.ce s is a set that is indeter.minate, it is there.fore Ms-con.stantly.

Pro.of:

This can be demonstrated usi.ng the suppos.ition and theo.rem (4-5).

Theo.rem (4.23): Let (ɼ), and let tp-s, such that eve.ry set s is an M-cl.osed set..Then:

If the matrix.:(ɼ,t)→(Ƴ,t') is sM-con.stante, and the vector.:(Ƴ,t')→(Ⱬ,t'') is Ms-con.stante, then the matr.ix:(g) is sM-con.stante, theref.ore:(g):(ƫ) is sM-con.stante, whi.ch is there.fore an s-indet.erminate set. If the matrix equa.tion (3.1) has an indeter.minate solu.tion, and the mat.rix func.tion (3.2) is Ms-con.tinuous, then the func.tion (3.3) is s-unso.lvable.

Pro.of:

This can be demons.trated by the hypot.heses and theo.rem 4.5.

Theo.rem (4.24): Let (ε,t), (ε,t'), and (ε,t'') be tp-s sets, whe.re t and t' sati.sfy that eve.ry s set is M-cl.osed. If g:(ε,t') is Ms-con.tinuous, and if g:(ε,t') → (ε,t'') is s-unsolvable, then g:(ε,t') → (Ⱬ,t'') is s-unso.lvable. , t^'') is SM-cons.tinuous.

Pro.of:

The pro.of is deri.ved from the premises directly.

Theo.rems (4.25): Let (ε, t), (ε, t'), and (Ⱬ, t'') be tp-s sets, and let ε and Ⱬ have the prop.erty that eve.ry s set is M-cl.osed. If the sys.tem has a solu.tion that invo.lves the follo.wing cha.nge of varia.bles: (ε, t) → (ε, t') is m-unso.lvable, and g: (ε, t') → (Ⱬ, t'') is Ms-con.tinuous, then g: (ε, t'') is sM-con.tinuous.

Pro.of:

The pro.of is deri.ved from the prem.ises dire.ctly.

Theo.rem (4.26): Let (ε, t), (ε, t'), and (μ, t'') be sets of tp-s, whe.re ε and μ sati.sfy the prop.erty that eve.ry set of s is M-cl.osed. If the mapp.ing ƒ:(ε, t) → (ε, t') is conti.nuous, and the mapp.ing g:(ε, t') → (μ, t'') is also conti.nuous, then g:(ε, t'') is sM-continuous, whi.ch implies that g:(ε, t) is also s-indet.erminate.

5. Spa.ced acr.oss the sky

Defin.ition (5.1): A set of tp-s.A is said to be conne.cted if it does not cont.ain the ss set A or the sw set B such that the inters.ection of the doma.ins of the two sets is emp.ty, that is, if U^(°s) and V^(°s) are both emp.ty.

Theo.rem (5.2): Let ε be a tp-s (ε, t) set that cont.ains a set A that is both a sw-.set and a set B that is a ss-set. Additi.onally, 〖A〗^(°s) ⊈ ¯B^s. Aft.er that, ε isn't conne.cted to the oth.er side.

Pro.of: Sin.ce 〖A〗^(°s) ⊈ ¯B^s, then 〖A〗^(°s) ⊈ 〖B〗^(°s) and ¯A^s ⊈ ¯B^s. Sin.ce A is a set of swa.ps and B^(°s) is an open set that cont.ains A^(°s), accor.ding to Theo.rem (3-18), A^(°s) ∩ B^(°s) = ∅. Sin.ce B is a set of ss's, and A^s is a clo.sed set, and A^s ⊈ B^s, by Coro.llay (3-19), $\overline{A} \cup \overline{B}$ = ε. As a res.ult, ε is not conne.cted to the oth.er end.

Theo.rem (5.3): Let tp be a set that cont.ains a sub.set of A, whe.re A is both a set of ss's and a set of sw's. After that, ε isn't conne.cted to the oth.er side.

Pro.of:

Let A be a set that is both an ss set and a sw set. As a res.ult, by Theo.rem 3-9, 〖A〗^c is a set that is both s-con.nected and sw-con.nected. It's obvious that A^(.c^(°s)))) is grea.ter than A^(°s). Then, by Theo.rem 3.16, 〖A^c〗^(°s) ∩ A^(°s) = ∅. If the set of indi.ces is partit.ioned into two sets, then the fir.st set, ¯(A.^c)^s, is equ.al to the sec.ond set, ¯A^s, whi.ch is contrad.ictory to the fact that A is an s-con.nected set. As a res.ult, accor.ding to the theo.rem ( ), ¯A^s ∪ ¯(A^.c)^s = ε. As a res.ult, ε is not conne.cted to the oth.er end.

Theo.rems (5.4): Let (ε, t) and (ε, t') be two tp-s, and let ƒ:(ε, t) ⟶(ε, t') be a bijec.tive, open, and conti.nuous func.tion. If ε is conne.cted via the swap chan.nel, then so is it conne.cted via the swap chan.nel as well.

Pro.of:

Supp.ose that the.re are sets A and B in ε that are both s- and sw-sets. Then 〖A〗^(°s) and ¯A^s are both sets that are emp.ty. As a res.ult, by Theo.rem 4.14, ƒ^(-1)(B) is an s-set in ε, and by Corol.lary 4.15, ƒ^(-1)(A) is a w-set in ε. Sin.ce ƒ is an s-o.pen set and conti.nues to be an s-cont.inuous set, there.fore (ƒ^.(-1)(A))^s + (ƒ^.(-1)(B))^s =ƒ^.(-1)(A) + ƒ^(-1)(B) =ƒ^.(-1)(Y) = Ӽ. Sin.ce ƒ is surje.ctive, open, and conti.nuous, we have 〖(ƒ^(-1)(A))〗^(°s) ∩ 〖(ƒ^(-1)(A))〗^(°s) = ƒ^(-1)(〖A〗^(°s)) ∩ ƒ^(-1)(〖B〗^(°s)) = ƒ^(-1)(〖A〗^(°s)) ∩ 〖B〗^(°s)) = ƒ^(-1)(∅) = ∅. As a res.ult, ε is not conne.cted to the oth.er end of the sti.ck, this contra.dicts the the.ory. As a res.ult, ε is conne.cted to the oth.er end.

Theo.rems (5.5): Let (ε, t) and (ε, t') be two tp sets, and let ƒ:(ε, t) ⟶(ε, t') be a bijective, clo.sed, and conti.nuous func.tion. If ε is conne.cted via a str.ing of len.gth l, then so is ε'.

Pro.of: Supp.ose that the.re are w-s.ets A and s-s.ets B in ε such that 〖(A〗^(°s)) ∩ 〖B〗^(°s) = ∅ and ¯A^s∪ ¯B^s = ε. As a res.ult, by Theo.rem 4.12, ƒ(B) is an ss-.set in ε, and by Corol.lary 4.13, ƒ(A) is an sw-.set in ε. Sin.ce ƒ is surje.ctive, clo.sed, and conti.nuous, ¯A^s∪ ¯B^s = ƒ(¯.A^s) ∪ ƒ(¯.B^s) = ƒ(¯.A^s) ∪ ¯B^s) = ƒ(Ӽ) = Ƴ. Sin.ce ƒ is surjective, clo.sed, and conti.nuous, 〖(ƒ(A))〗^° and 〖(ƒ(A))〗^(°s) also fol.low. = ƒ(〖A〗^(°s)) ∩ ƒ(〖B〗^(°s)) = ƒ(〖A〗^(°s) ∩ 〖B〗^(°s)) = ƒ(∅) = ∅. As a res.ult, Ƴ is not conne.cted to the oth.er side, this contra.dicts the the.ory. As a res.ult, we can ded.uce that Ӽ is conne.cted to the sw-.net.

6- Conclusions:

We keep in mind the ide.as of s-mi.nimal open and s-ma.ximal clo.sed sets in this stu.dy. We pres.ent and invest.igate new typ.es of sets that arise from the.se ide.as, such as s-st.rong and s-weak sets. Characte.ristics and prope.rties of the new typ.es of sets. This Fut.ure research will now have a new ave.nue to invest.igate how the conc.epts of s-st.rong and s-w.eak sets are used.

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