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Mathematical Modeling of The Vertical Launch and Operation of An Aircraft Missile Gyroscope

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Abstract:
The vertical launch system has been widely adopted to land- and sea-based guided missile systems, because of its excellent operation flexibility, quick response capability and effective utilization of the space volume over the deck. In this paper the advantages and technical characteristics of the vertical missile launch systems are presented and it is analyzed a process of inclination (turning) with respect to a missile necessary for an entrance in guidance trajectory. Special emphasis is placed on the role of the gyroscopic control in maintaining stable angle motion during important phases of flight initiation.
A mathematical model is developed for the motion of a guided missile that takes place between vertical launch and turn on torque. The model considers the forces and moments acting on a missile control system, control forces generated by impulse thrusters, inertia conditions, mass change as a result of fuel depletion and location of points of application. In addition, the angular response of a three directional gyroscope subjected to fixed and linearly moving bases is mathematically represented with generalized coordinates.
The derived equations include extra damping and restoring moment due to aerodynamic pressure, gravitational effect of the ground terrain, and external disturbances. Both correctable and uncorrectable gyro spin configurations are examined, so that the model may be used over the full envelope of missile guidance platforms. The formulated identifiability conditions establish a solid foundation for the structure design, which can be incorporated in authors' future research on intelligent missile control system algorithms to promote guidance accuracy, stability and survivability in the stages of vertical launch and initial maneuvering.

Keywords: missile, flight, vertical launch, angular rotation.

1. Introduction

Vertical launching of guided missiles for ships offers faster engagement upon any target bearing and in addition can provide high firing rates with a reduced amount of ammunition on-board [1], [2]. Vertical launch systems -VLS- have a number of advantages, including for instance reducing the radar cross section of ships and not requiring time to open and close hatches or mechanisms to move missiles between deck and magazine such as in a below-deck launcher: each missile enters its own cell, where one guided missile system (GMS) is a "container" consisting of tube (or pipe), hatch / door and contents). VLS pucks are/could be developed to fire different GMs (ASM, AAW or ASW in other words) and make the carrier multifunction according to mission fitted. It has demonstrated that such VLSs can be modernized to deploy and launch more advanced GMs. But the danger of firing from below-deck outweighs the positive during launch and with respect to directing the GM in that direction [3]. The path of a vert There are two types of vertical launch: - launching under its own engine power (hot launch); - using a catapult to eject the missile from the launch container (cold launch).

We will assume that the steering (turning) of the GM is made under the action on it of control moment formed by effort Py at controllable actuators (block impulsive thuster). With tensioning force, the deflection time of GM to a desired angle is dependent on the point of application and control forces [4]. The parameters at the completion of the turning segment become the initial parameters for guidance trajectory and they ultimately determine its quality.

2. Materials and Methods

Let's consider a mathematical model of the GM's motion before the start of the insertion phase onto the guidance trajectory.

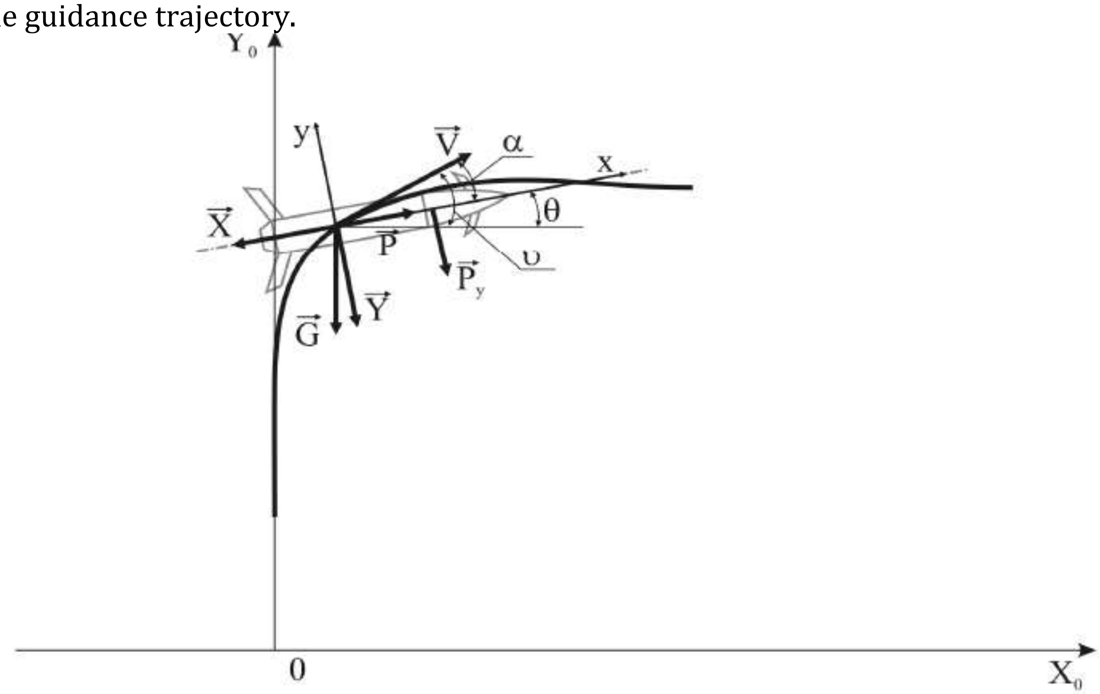

Figure 1. Diagram of the forces acting on the GM during a torque turn

The motion of the missile control system, according to the diagram of forces acting on the aircraft shown in the figure, is described by the following system of equations [2]:

$$m(t)\dot{V} = -X_a(M)\cos\alpha - m(t)g\sin\Theta - Y_a\sin\alpha,$$

$$m(t)V\dot{\Theta} = -P_y\cos\alpha - m(t)g\cos\Theta + X_a\sin\alpha,$$

$$m(t) = m_0 - m_c(t_a)t, \quad X_a(M) = \frac{(\rho V^2)}{2} S_м C_{X_a}(M),$$

$$\ddot{\vartheta} = \frac{P_y(x_{цм}-x_p)}{I_{zz}}), \alpha = \vartheta - \Theta,$$

where $x_{цм}$ is the coordinate of the GM's control system's center of mass; x_p is the coordinate of the point of application of the missile control force (the coordinate of the control system's center of gravity); I_{zz} is the moment of inertia of the missile control system relative to the equatorial axis passing through its center of mass; m_0 is the total mass of the missile with fuel; $m_c(t_a)$ is the fuel mass flow rate per second.

Now we examine the basing of the missile control system on the internal part of the aircraft. We will consider the operation of the upper organizational layer (the layer for decision-making based on the analysis of external situations) of a multi-level intelligent control system for a homing missile.

3. Results

Figure 2 shows a schematic diagram of a gyroscope according to scheme A1, illustrating the relative positions of the elements. Axis 6, to which the internal EO is connected, can be rigidly attached to the housing or mounted in bearings. The axis can rotate at a velocity w, and the rotor at a velocity Ω, around the zp axis of the associated X_p, Y_p, Z_p coordinate system.

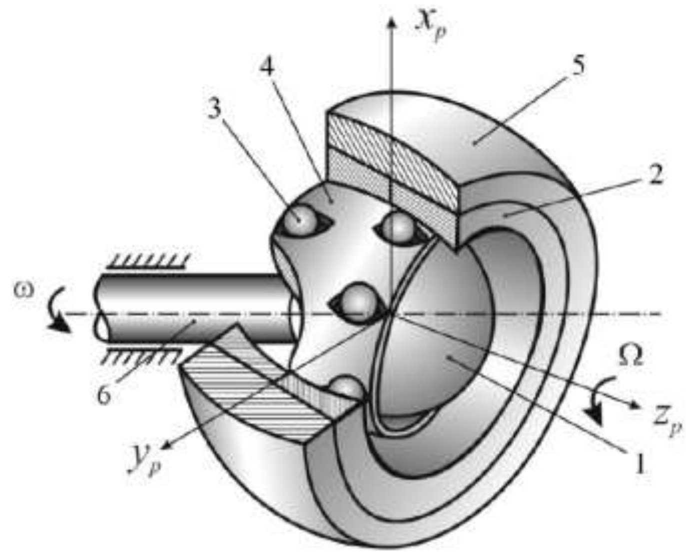

Figure 2. Schematic diagram of a three-degree gyroscope

Let's consider the torque rotation achieved by using a pulse motor unit as control elements (Fig. 1).

Based on the gyroscope, uncorrectable [5] and correctable [6, 7] gyroscopic devices for operation as part of a gyroplatform have been created. A correctable gyroscopic device, in which the rotor is mounted relative to bearings, has an additional degree of freedom, is also designed based on the A1 scheme. The dynamic characteristics of correctable and uncorrectable gyroscopes with a PPC are due to the presence of specific cross and radial restoring moments. The *XYZ* coordinate system (CS) in Fig. 4 is associated with the housing, and the *xyz* CS, whose position is determined by angles a and b, is associated with the main axis of the gyroscope.

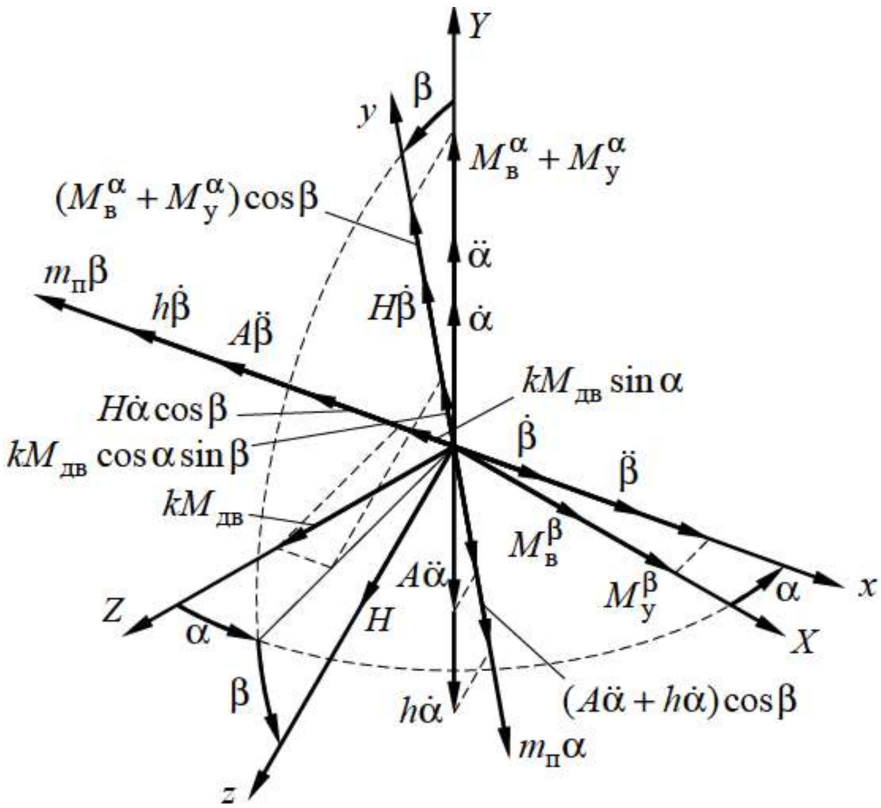

Figure 3. Calculation scheme for deriving the equations of motion of a gyroscope

Using the above calculation scheme, we mathematically model the motion of a gyroscope rotor on a fixed base known [8] and taking into account the motion of the base with angular velocities, the projection of the absolute angular velocity of an aircraft, in particular an air combat missile, on its main axes [9], denoted as ω_{ox}, ω_{oy}, ω_{oz} can be written as

$$\begin{cases} A(\ddot{\alpha} + \dot{\omega}_{0Y}) + (h_{vn} + h_{sdv} + h_{tg} + h_{sd})\dot{\alpha} + m_{\Pi}\alpha - C\dot{\varphi}(\omega_{0X} + \dot{\beta} - \omega_{0Z}\alpha) - m_{pB}\beta = \\ \mathrm{M}_{t}^{\alpha} - \mathrm{M}_{sdv}^{\alpha} - \mathrm{M}_{tg}^{\alpha} - \mathrm{M}_{sd}^{\alpha}, \\ A(\ddot{\beta} + \dot{\omega}_{0X}) + (h_{vn} + h_{sdv} + h_{tg} + h_{sd})\dot{\beta} + m_{\Pi}\alpha - C\dot{\varphi}(\omega_{0Z} + \omega_{0Y+}\dot{\alpha}) - m_{pB}\alpha = \\ \mathrm{M}_{t}^{\beta} - \mathrm{M}_{sdv}^{\beta} - \mathrm{M}_{tg}^{\beta} - \mathrm{M}_{sd}^{\beta}, \\ C\dfrac{d^{2}\varphi}{dt^{2}} = M_{d} - M_{n}, \\ (\mathrm{A}_{X} + \mathrm{B}_{X} + \mathrm{C}_{X})\dot{\omega}_{0X}^{2} - (k_{\alpha} + k_{\beta}) = 0, \\ (\mathrm{A}_{Y} + \mathrm{B}_{Y} + \mathrm{C}_{Y})\dot{\omega}_{0Y}^{2} - (k_{\alpha} + k_{\beta}) = 0 \end{cases}$$

Here A is the equatorial moment of inertia of the rotor; A_X, A_Y, B_X, B_Y, C_X, C_Y are the moments of inertia of the suspension ring relative to the principal axes of inertia along the corresponding axes; h is the damping coefficient; m_{Π} is the specific cross moment; C is the axial moment of inertia of the rotor; φ is the angle of rotation of the gyroscope rotor; $\varphi^{\cdot}$ is the angular velocity of the gyroscope rotor [11]; M_d, M_n are the electromagnetic moment and the load moment reduced to the motor shaft, respectively; m_{pB} is the specific radial restoring moment; α, β are the generalized coordinates of the rotor motion; it is proposed to introduce new terms of the equation [12]: h_{vn} is the damping coefficient of internal force loads; h_{sdv} is the damping coefficient of the air pressure force; h_{tg} is the damping coefficient of the gravity of the mountain and different spatial volumetric elements; h_{sd} is the damping coefficient of high-density snow and rain [13], [14]; $\mathrm{M}_{sdv}^{\alpha}$ is the moment of air pressure; M_{tg}^{α} is the gravitational moment of the mountain and various spatial volumetric elements; M_{sd}^{α} is the moment of high-density snow and rain [15].

4. Conclusion

In this paper, a complete mathematical model that represents the vertical launch and the action of gyroscopic stabilisation of an air-launched combat missile is constructed. The motion of the missile during lift off and turning was examined and impact of control forces, variation in mass, inertial parameters on angular motion of missile are discussed. The motion variables at the end of turning process were found to have significant impacts on following guidance trajectory.

A comprehensive mathematical formulation of the gyroscope rotor motion on a stationary and moving platform was discussed. The obtained system of equations describes besides classical gyroscopic effects also other damping and restoring moments due, e.g., to aerodynamic forces, the gravitational field of an irregular terrain or external conditions. This greatly improves the applicability of the model to actual combat.

The results show that the model can be successfully utilized for design and optimization of intelligent multi-level missile control systems. Implementation of the derived equations in algorithms allows the missile maneuverability, angular stability to be raised and the chances of interception with an enemy interceptor rocket avoiding. The general method developed in this paper provides a theoretical reference for future work with guided missile dynamics, gyroscopic control system and new vertical launching technique.

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