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Fractal Analysis of Blood Flow Based on Heart Valve Dynamics

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Abstract:

Introduction. This article develops a mathematical model and algorithms of blood flow taking into account the dynamics of heart valves. The main goal of the research is to determine the nonlinear and complex dynamics of heart activity.

Methods. The model considers the heart chambers as elastic reservoirs that can deform to a certain volume. A point dynamic model is used to represent the dynamics of blood flow. The movement of the valves follows a specific dynamic equation, and valve function is expressed as a smoothly transitioning monotonic function. The complex dynamic characteristics of heart signals are determined using the MFDFA (Multifractal Detrended Fluctuation Analysis) method..

Results. The proposed model allows for the differential analysis of healthy and pathological signals. The main advantages of the model are: low demand for computational resources, a relatively small number of parameters, and the fact that most of them are practically measurable quantities. The results were implemented in the Python environment and the solutions were presented in a visual format.

Conclusion. The mathematical model has made it possible to gain a deeper understanding of the normal and pathological states of the heart. This, in turn, provides practical assistance in developing new diagnostic and treatment methods in medicine. The parameters of the model can be assessed in medical practice through indirectly measurable quantities (dynamics of chamber volume, ejection fraction, and changes in pressures).

Keywords: HRV, mitral valve, multifractal analysis, MFDFA, heart signals, visualization.

Introduction

The human heart consists of four chambers. The chambers are connected to each other in pairs: the left atrium - the left ventricle and the right atrium - the right ventricle. These pairs have a similar structure and in a healthy state there is no direct connection between them. Therefore, the heart model consists of a system of equations that are identical for the left and right halves, which differ only in the values of the parameters. This article presents a system of equations for the left half of the heart.

The dynamics of heartbeats is a complex system, like the human body, which provides information about the cardiac system. It is considered one of the leading areas of modern medicine and is studied in the department of cardiology. In this area, many methods for analyzing heartbeats have been developed and are currently being used [1].

In recent years, digital signal analysis methods, in particular multifractal modeling, have become increasingly important in the assessment of cardiac function. While traditional methods analyze linear components of cardiac signals, the multifractal approach allows for the measurement of nonlinear and scale-dependent complexity[5]. The aim of this study is to develop a mathematical model of blood flow and an algorithm that takes into account the dynamics of valves in the heart, and based on this, to determine the nonlinear and complex dynamics of cardiac function.

RESEARCH PROBLEM STATEMENT

To create a mathematical model of heart function, taking into account the dynamics of heart valves and their effect on blood flow, and to study the possibilities of differentiating various physiological states, including healthy, stressed, and pathological states, using this model. The research aims to solve the following problems:

1. Dynamics of heart valves and blood flow: Modeling the opening and closing of heart valves, the flow rate between the valves, and the control parameters in the system. Describing these processes using mathematical formulas and reflecting changes over time.
2. Multifractal analysis and scaling indicators: Determination of scaling indices ($\zeta(q)$, $\tau(q)$, $h(q)$, $f(\alpha)$) for multifractal description of valve dynamics. Based on these indices, uncertainties in the motion of heart valves and blood flow system, as well as the complexity of the system, are analyzed.
3. Differentiating physiological states: Evaluation of parameters to differentiate between resting, stressed, and pathological states by analyzing the movements of the heart valves and blood flow. This is particularly helpful in understanding the mechanical movement of the heart valves and changes in blood flow velocity.
4. Diagnostics based on optimal parameters: Determining the optimal set of parameters in a mathematical model of blood flow and making diagnostic conclusions. For example, making clinical diagnoses by optimizing parameters such as valve opening angle, flow velocity, elasticity and resistance coefficients.

The goal of the research is to optimize the parameters necessary to detect various physiological conditions and automatically draw diagnostic conclusions by mathematically modeling the complexities of the heart valve movement and blood flow system. The main advantages of the model are to make clinical analysis more accurate and efficient, minimize computational resources, and speed up the diagnostic process.

Materials & Methods. The concept of multifractal analysis was first developed by B. Mandelbrot in the 1980s, and when applied to biophysical signals (heart, EEG, EMG), the time-dependent change in fractal dimension represents the healthy or pathological state of a physiological system [1].

The MFDFA algorithm proposed by EJ Kantelhardt (2002) has been adopted as a basic method in biological signal analysis[2].

In recent years, a number of researchers have investigated the possibility of detecting heart rate variability, stress states, and arrhythmias using multifractal spectra of ECG signals [2–4]. However, the issue of visualization — diagnostics using multifractal and thermal images and spectra, i.e., patient diagnosis — has not been fully automated. Therefore, the article explores the construction of a mathematical model of the multifractal dynamics of the heartbeat and the automation based on this model.

The layout of the chambers and the location of the valves of the left heart are shown in Figure 1. The left atrium receives the pulmonary veins, while the left ventricle pumps blood into the aorta. The valve between the atrium and the ventricle is called the mitral valve. The valve between the ventricle and the aorta is called the aortic valve.

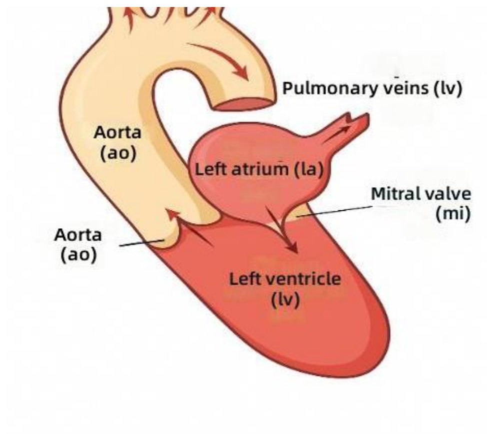

Figure 1. Schematic of a point model of the left heart

The article considers the heart chambers as elastic reservoirs with a deformable volume. Their elasticity depends on the dynamics of potentials propagating through the myocardium. The flow between the atria and ventricles, as well as between the ventricles and the aorta, depends on the opening angle of the corresponding valves.

Table 1. Descriptions of the indices representing the blood flow model in the heart

Index	Description
d	diastole
fr	friction force
max	maximum value
thousand	minimum value
p	compressive strength
pb	The beginning of the P-wave
pw	P-wave duration
r	resistance force
s	systole
s1	beginning of systole
s2	end of systole
is	mitral valve

oh	aortic valve
lpv	outlet of the left pulmonary artery (entrance – to the left atrium)
sound	entrance to the aorta (left ventricular outlet, aortic sinus)

The article described the dynamics of blood flow within each chamber of the heart using a point-dynamic model:

$$I_k \frac{d^2V_k}{dt^2} + R_k \frac{dV_k}{dt} + E_k(t)(V_k - V_k^0) + P_k^0 = P_e, \qquad k \in \{la, lv\}, \quad (1)$$

here $k-$ heart chamber indices (left atrium) $-la$, left ventricle $-lv$, $V_k^0 -$ resting (initial) volume of the heart chambers, resting pressure $P^0 -$ This is the pressure value taken as the zero point. Here $I-$ inertia coefficient of the chamber wall, $R-$ is the hydraulic resistance coefficient of the chamber. Equation (1) can be considered as a modification of model [3]. In this model, the elasticity coefficient of the heart valve chamber wall is assumed to be time-dependent. That is, $E_k(k)-$ The coefficient of elasticity of the chamber wall. A "chamber" is a chamber or ventricle of the heart, generally speaking, the cavity where blood collects. Its wall is stretchy, elastic, and how stiff or soft it is is expressed by a special number, and this coefficient of elasticity E is equal to. Usually this model E may be permanently fixed (for example, $E - const$). In the study, the elasticity coefficient was assumed to be time-dependent. The heart chamber is not only under pressure from the blood inside, but also from the outside, i.e. from the chest cavity, pericardium, etc. Until then, the external $P^0 -$ defined as [4,6]. In the now proposed model: the elasticity of the wall depends not only on the volume V, It is being researched that it may also be due to this external pressure. $E(t)(V - P^0)$ $-$The meaning of the expression: $E(t) -$ time-dependent elasticity, $V -$ volume inside the chamber, $P-$ constant external pressure.

Based on the concept of variable elasticity, $E(t)$ The time evolution of the function can be given as:

$$E(t) = E^d + \frac{E^s - E^d}{2} e(t), \qquad 0 \le e(t) \le 1, \quad (2)$$

given in this form, that is, the activation function $e(t)$ is a recurring (periodic) function throughout the entire period of the heart's activity. It is given separately for each chamber depending on its systole-diastole duration. Each chamber of the heart (left atrium, right atrium, left ventricle, right ventricle) works differently during systole and diastole, i.e. systole is the contraction phase of the chamber, diastole is the relaxation and filling phase. Since the duration of systole and diastole is different for each heart chamber, their elasticity coefficient or pressure function is different in the model. That is: the systole of the left ventricle may be longer and the diastole of the right atrium is different, the elastic phase in the left ventricle is softer, shorter, etc. Therefore, in the mathematical model: $E_{LV}(t)$, $E_{LA}(t)$, $E_{RV}(t)$, $E_{LA}(t)$ Four separate time-dependent functions are given, such as: . It cannot be expressed with one general function, because the mechanical function of each camera is different.

The activation function for the left ventricle is defined as:

$$e_{lv}(t)=\begin{cases}1-\cos\left(\dfrac{\pi t}{T_{s1}}\right), & 0\le t\le T_{s1},\\ 1-\cos\left(\dfrac{\pi(T_{s2}-t)}{T_{s2}-T_{s1}}\right), & T_{s1}\le t\le T_{s2},\\ 0, & T_{s2}\le t\le T,\end{cases} \qquad (3)$$

Here $T_{s1}-$ duration of the initial phase of systole, $T_{s2}-$ end of systole, $T-$ the duration of a complete cardiac cycle.

The activation function for the left-hand side is given by:

$$e_{la}(t)=\begin{cases}0, & 0\le t\le T_{pb},\\ 1-\cos\left(\dfrac{\pi(t-T_{pb})}{T_{pw}}\right), & T_{pb}\le t\le T_{pb}+T_{pw},\\ 0, & T_{pb}+T_{pw}\le t\le T,\end{cases} \qquad (4)$$

here $T_{pb}-$ the beginning of the wave, $T_{pw}-$ wave duration.

The left ventricular ejection fraction varies with heart rate (HR). From this result, in the activation functions (3) and (4) T_{s1}, T_{s2}, T_{pb}, T_{pw} can be used by changing parameters such as

The law of conservation of mass states that the total mass of matter in a closed system does not change, that is, for each chamber $\dfrac{dV}{dt}=\Delta Q$ is written as follows:

$$\frac{dV_{la}}{dt}=Q_{lpv}-Q_{mi}\ ,\ \frac{dV_{lv}}{dt}=Q_{mi}-Q_{ao}\ ,(5)$$

here: $Q_{lpv}-$ flow into the left atrium from the pulmonary veins, $Q_{mi}-$ flow from the atrium to the ventricle through the mitral valve, $Q_{ao}-$ The flow through the left ventricle into the aorta is shown in Figure 1. The loss of momentum in the valves connecting the heart chambers to each other and to the outside is the valve opening angle θ_k Poiseuille's law depending on $Q=\dfrac{\pi r^4(P_1-P_2)}{8\eta L}$ is determined based on. In this formula $Q-$volumetric flow rate (ml/s or L/min), $r-$vessel radius, $(P_1-P_2)-$pressure difference, $\eta-$viscosity, $L-$is the length of the vessel. A negative value of the current indicates that the direction has reversed.

$$Q_{ao}=S_{ao}(\theta_{ao})\frac{P_{lv}-P_{sas}}{R_{ao}}\ ,\ Q_{mi}=S_{mi}(\theta_{mi})\frac{P_{la}-P_{lv}}{R_{mi}}\ ,\ Q_{lpv}=\frac{P_{lpv}-P_{la}}{R_{lpv}}\ ,(6)$$

In this $S_{ao}(\theta_{ao})-$ aortic valve opening area, $S_{mi}(\theta_{mi})-$ mitral valve opening area, R_{ao}, R_{mi}, R_{lpv} $-$aortic, mitral and pulmonary vein resistances, respectively. For the valve opening angle $g(\theta)$ is given as follows:

$$g(\theta)=\begin{cases} \theta^{\theta_{\min}}, & 0\le\theta\le\theta_{\max}, \\ 0, & \theta<0 \ \ yoki\ \theta>\theta_1, \end{cases} \quad (7)$$

This function ensures a smooth change in the valve opening area. In the work, it was assumed to be a function of the valve density.

$$S(\theta)=S_{\max}\,g(\theta), \quad (8)$$

Here $S_{\max}$ – maximum opening area of the valve.

In this case, the valve function $g(\theta)$ changes its state at a given time interval and, according to the model in [3], has the following form:

$$g(\theta)=\begin{cases} 0, & 0\le t<T_{open}, \\ 1, & T_{open}\le t<T_{close}, \\ 0, & T_{close}\le t<T, \end{cases} \quad (10)$$

In that case $T_{open}^{ao}=0.15\ s$, $T_{close}^{ao}=0.33\ s$, $T_{open}^{mi}=0.44\ s$, $T_{close}^{mi}=1\ s$, The model studied does not take into account some physiological processes. For example, the flow back to the ventricle or to the aorta at the end of systole (regurgitation) is not taken into account. It is also difficult to adapt the model to different heart rate conditions (HR).

In the study, it was assumed that the time interval during which the valve position changes is not very small, that is, the valve opening and closing do not occur in a short time. Therefore, the valve motion is subject to a certain dynamic equation and the valve function is expressed as a smoothly passing monotonic function.

$$g(\theta_k)=\begin{cases} 0, & 0\le\theta_k<\theta_k^{\min}, \\ \dfrac{1-\cos\left(\dfrac{\pi(\theta_k-\theta_k^{\min})}{\theta_k^{\max}-\theta_k^{\min}}\right)}{2}, & \theta_k^{\min}\le\theta_k<\theta_k^{\max}, \quad k\in\{ao,mi\} \\ 0, & \theta_k>\theta_k^{\max}, \end{cases} \quad (11)$$

In this $\theta_k^{\min}$ – the angle at which the valve starts to open, $\theta_k^{\max}$ – The angle at which the valve is fully opened. The equations of motion of the aortic and mitral valves are determined based on Newton's second law of action potential propagation.

The proposed point model is based on physical principles to represent the dynamics of blood flow in the heart. This model allows for physiologically adapted calculations both under normal conditions of the heart and in some pathological conditions. The main advantages of this approach are: a) low demand for computational resources, b) relatively small number of parameters, c) the fact that most of them are practically measurable quantities in large cardiology centers.

Valve function $g(\theta)$ has a significant impact on cardiac dynamics. The model based on the assumption of short-term opening or closing of the valves gives a simplified mathematical model, formula 11. Taking this into account, the parameters of this model, i.e. the timing of valve opening and closing, can be determined based on experimental data.

RESULTS

Current echocardiographic studies of short-term valve position changes do not match physiological observations. These studies lead to a loss of stability in the calculation process and, as a result, the appearance of false physiological artifacts. Therefore, a model that does not rely on short-term position changes provides results that are closer to physiological data.

Table 2. Model coefficients

Parameter	Value	Parameter	Value
E_{lv}^{max}	2.0 mmHg/ml	θ_{ao}^{min}	0°
E_{lv}^{min}	0.5 mm.rt.st./ml	θ_{ao}^{max}	75°
R_{lv}	10−5·5 mm.rt.st./cm/ml	θ_{min}^{max}	75°
E_{la}^{max}	0.25 mm.rt.st./ml	P_{lv}	100 mm.rt.st.
R_{la}	10−5·5 mm.rt.st./cm/ml	T_{s1}	0.3 c
T_{pv}	0.8 c	T_{s2}	0.44 c
K_{ao}^{max}	5.5·103 rad/c·mm.rt.st.	T_{pb}	0.5 c
K_{ao}^{min}	5.5·103 rad/c·mm.rt.st.	K_{ao}^{f}	0.5 rad/(c ml)
K_{mi}^{max}	5.5·103 rad/c·mm.rt.st.	K_{min}^{f}	0.5 rad/(c ml)
K_{mi}^{min}	5.5·103 rad/c·mm.rt.st.		

In medical practice, however, these parameters must be determined indirectly, that is, they are estimated through the correlation of the following measured quantities:

a) camera volume dynamics;
b) cardiac output and cardiac output;
c) pressure changes and other physiological indicators.

Measurement of some parameters of the model (inertia coefficients, hydraulic resistances, elastic modulus) can only be carried out within the framework of special clinical studies, based on previously developed methodologies.

CONCLUSION

The article presents a mathematical approach based on modeling blood flow in the left ventricle of the human heart. The article shows that modeling blood flow and heartbeat using multifractal analysis, taking into account the dynamics of each chamber and valve of the heart, and the correct representation of the flow between the valves and chambers, as well as their elasticity and time-dependent motion, is one of the key factors in creating an accurate model of heart function.

In general, this article is a scientific study on the development of an approach aimed at complex and complete modeling of heart beats and blood flow. Through the mathematical model, a better understanding of normal and pathological states of the heart has been created, which will provide practical assistance in the development of new diagnostic and treatment methods in medicine.

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