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# DETERMINATION OF ANALYTICAL DEPENDENCE AND NUMERICAL CALCULATION OF TORQUE AND DISPLACEMENT OF VIBRATION OF THE SUPPORT OF AN INTERNAL COMBUSTION ENGINE

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**Abstract:**

This article examines the analytical determination of torque and numerical calculation of vibration displacement of a mobile vehicle's internal combustion engine mount. An analytical relationship between engine torque, mount stiffness, damping coefficient, and vibration displacement is developed. Key mathematical modeling methods based on differential equations of motion, numerical solution methods, and computer simulation are presented. The results obtained allow for improved accuracy in predicting dynamic loads, reducing engine vibration levels, increasing the service life of mounts, and enhancing the reliability of mobile vehicles.

**Keywords:** internal combustion engine, engine support, vibration, torque, numerical calculation, dynamic loads, rigidity, damping.

## Introduction

Modern mobile machines operate under significant dynamic loads caused by the uneven operation of the internal combustion engine, changing road conditions, and external disturbances. One of the main sources of vibration is variable engine torque, which causes vibrations in the powertrain and transfers vibration to the machine's structural elements. Therefore, studying the analytical relationship between engine torque and the vibrational displacement of its supports is a pressing scientific challenge [1].

I.I.Artobolevsky's work examines the fundamentals of machine dynamics, methods for determining inertial forces, and the calculation of oscillatory processes in mechanical systems. The author notes that dynamic engine loading depends significantly on changes in torque and the characteristics of the elastic elements of the engine mount [2].

V.A. Kudryavtsev's research focuses on vibration analysis for transport vehicles. It has been shown that vibration amplitude reduction is achieved by optimally selecting the stiffness and damping coefficients of rubber-to-metal engine mounts [3].

A.A. Alexandrov's work explores methods for mathematical modeling of dynamic processes in mechanical systems. The author proposes using second-order differential equations to model the spatial oscillations of power units in mobile machines [4].

According to Yu. I. Borisov **,** unevenness of the engine operating cycle leads to the occurrence of harmonic disturbing moments, which significantly affect the durability of engine mounts and transmission connecting elements [5].

Research by N.A. Makhutov shows that cyclic vibration loads are one of the main causes of fatigue damage accumulation in metal machine structures. The use of analytical models allows for predicting the service life of components already at the design stage [6].

V.V. Biryukov's work presents the results of an experimental study of vehicle engine vibration isolation. The author found that the use of modern damping materials can reduce vibration acceleration by 25–40% .

S.V. Belov's research focuses on numerical methods for solving machine dynamics problems. It has been noted that the use of the finite element method ensures high accuracy in determining natural frequencies and engine mount displacements [7].

E.A. Larin's work examines modern methods for computer modeling of vibration processes using ANSYS and MATLAB software packages. The resulting models allow one to determine the relationship between changes in engine torque and the amplitude of vibrational displacement of supports under various operating conditions [8].

Thus, an analysis of the literature reveals that existing studies are primarily focused on experimentally determining engine vibration characteristics, while the analytical relationship between engine torque and the vibrational displacement of its mount requires further refinement. Therefore, the aim of this study is to develop an analytical model and perform a numerical calculation of the dynamic behavior of an internal combustion engine mount [9].

## Methodology

The study is based on theoretical mechanics, machine dynamics, mathematical modeling, and numerical analysis. The engine is considered as a concentrated mass mounted on elastic-damping supports.

For the numerical calculation, methods of integrating differential equations and computational analysis of changes in system parameters for different values of engine torque, support rigidity and damping coefficient were used.

## Result And Discussion

The main design parameters of the crank mechanism are the diameter $D$ and the piston stroke $S$ , as well as their ratio $S / D$ and the crankshaft crank radius $R = S / 2$. For tractor engines, the ratio $S / D$ is in the range from 0.9 to 1.2, for automobile engines $S / D = 0.8 ... 1.05$. Engines in which the piston stroke is less than its diameter are called short-stroke ; otherwise, the engine is called long-stroke [10] .

$R$ / $L$ is also used to describe the crank mechanism of an internal combustion engine , where $L$ is the connecting rod length. For automobile and tractor engines, this parameter ranges from 0.23 to 0.30 [11].

The diagram of the central crank mechanism is shown in Figure 1. The diagram of the central crank mechanism shows: φ – the angle of rotation of the crank at a given moment in time, measured from the cylinder axis in the direction of rotation of the crankshaft; β – the angle of deflection of the connecting rod in the plane of its rolling; $S$ – the piston stroke, equal to two crank radii: $S = C'C'' = 2R$ ; $R$ – the radius of the crank, $R$ = OA; $L$ – the length of the connecting rod, $L$ = CA = $R$ /λ; ω – the angular velocity of the crankshaft.

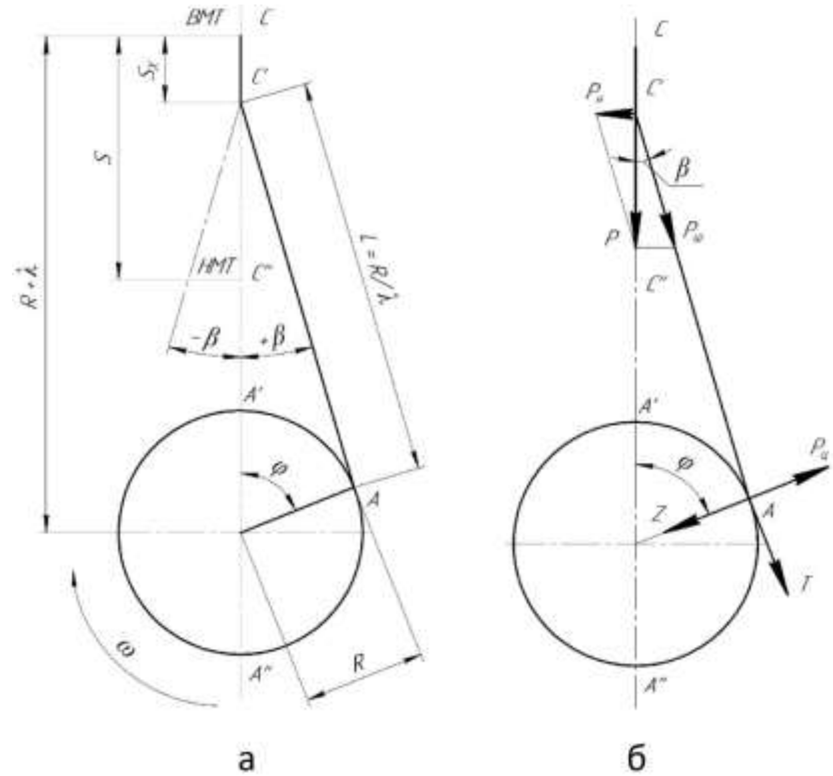


**Figure 1 – Diagrams of the central crank mechanism: *a* – for kinematic calculation; *b* – for dynamic calculation.**

At φ = 0 (the reference point), the piston is at top dead center (point $C$ ), and the crank is in position OA'. At φ = 180 °, the piston is at bottom dead center (point $C$ ''), and the crank is in position OA''. Each stroke of the cycle consists of the piston moving from top dead center to bottom dead center or vice versa.

We find the engine torque in another way [9] by applying a force $T^1$ to the crankshaft axis. and $T^2$ , equal and parallel to the force $T$ . Then the forces $T^1$ and $T^2$ form a pair of forces with a moment equal to

$$M_{kr} = T\,R \qquad (1)$$

where T – the component of the force tangential to the crank radius circle, directed along the connecting rod axis; R – crank radius.

The force $P_{sh}$ directed along the axis of the connecting rod is decomposed into two components: tangential and tangent to the circle of the crank radius

$$T = P_{sh} sin(\varphi + \beta), \qquad (2)$$

$$Z = P_{sh} cos(\varphi + \beta), \qquad (3)$$

Where $P_{sh}$ – force directed along the connecting rod axis, H ; φ – crank rotation angle at a given moment in time, deg ; β – connecting rod deflection angle in the plane of its rolling, deg.

The force directed along the connecting rod axis is determined by the formula

$$P_{sh} = Ptg\beta, \qquad (4)$$

The total force acting along the axis of the cylinder will be equal to

$$P = P_r + P_j, \qquad (5)$$

where $P_r$– gas pressure force in the engine cylinder , $N$.
We insert equations (1-5) in order

$$M_{kr} = \left(P_{sh} sin(\varphi+\beta)\right) R = \left(Ptg\beta\ sin(\varphi+\beta)\right) R = \left(\left(P_r+P_j\right) tg\beta\ sin(\varphi+\beta)\right) R, \quad (6)$$

The inertial force from the reciprocating motion of the crank mechanism is calculated using the formula

$$P_j = -m_j R\omega^2 (cos\varphi + \lambda\ cos2\varphi), \quad (7)$$

where $m_j$is the mass performing reciprocating motion and concentrated along the piston ring axis, kg; $R$ is the crank radius, m; ω is the angular velocity of the crankshaft, $min^{-1}$ ; φ is the crank rotation angle at a given moment in time, deg; λ is the ratio of the crank length to the connecting rod length [12], [13].
The mass performing a reciprocating motion and concentrated along the axis of the piston ring is calculated using the formula:

$$m_j = m_n + (0{,}2\ldots0{,}3) m_{sh}, \quad (8)$$

Where $m_n$– weight of piston group, kg ; $m_{sh}$– weight of connecting rod group, kg .
Then the gas pressure forces in the engine cylinder are calculated as

$$P_r = p_r F_n, \quad (9)$$

where $F_n$– piston area, $m^2$ .
To simplify the dynamic calculation, the gas pressure forces acting on the piston area are replaced by a single force directed along the cylinder axis.
The excess pressure of gases in the engine cylinder is determined by the formula $p_r = p_i - p_o$ (10)
where $p_i$– gas pressure in the cylinder, Pa; $p_o$– atmospheric pressure, Pa .
We insert equations (7-10) in order into (6)

$$M_{kr} = \left(\left(P_r + \left((-m_n + (0{,}2\ldots0{,}3) m_{sh}) R\omega^2 (cos\varphi + \lambda\ cos2\varphi)\right)\right) tg\beta\ sin(\varphi+\beta)\right) R = \left(\left((p_r F_n) + \left((-m_n + (0{,}2\ldots0{,}3)(m_n + (0{,}2\ldots0{,}3) m_{sh})\right) R\omega^2 (cos\varphi + \lambda\ cos2\varphi)\right)\right) tg\beta\ sin(\varphi+\beta)\right) R = \left(\left(((p_i - p_o) F_n) + \left((-m_n + (0{,}2\ldots0{,}3)(m_n + (0{,}2\ldots0{,}3) m_{sh})\right) R\omega^2 (cos\varphi + \lambda\ cos2\varphi)\right)\right) tg\beta\ sin(\varphi+\beta)\right) R.$$

$$M_{kr} = \left(\left(((p_i - p_o) F_n) + \left((-m_n + (0{,}2\ldots0{,}3)(m_n + (0{,}2\ldots0{,}3) m_{sh})\right) R\omega^2 (cos\varphi + \lambda\ cos2\varphi)\right)\right) tg\beta\ sin(\varphi+\beta)\right) R. \quad (11)$$

In the work of Yu.V.Khromov, the engine is defined by partial speed characteristics and partial regulatory. For the dynamic system under consideration, the engine operation modeling as a function of torque versus rotational speed was performed using the regulatory characteristic. The engine torque on the regulatory and corrective branches of the characteristic as a function of rotational speed ($n$) was determined using the following dependencies [14], [15].

$$M_{kr} = M_{kr\ emax} - (n/n_x - \alpha/(1-\alpha))^2 (M_{kr\ emax} - M_{kr\ en})$$

where $n_x$ – engine idle speed; $M_{kren}$ – nominal engine torque; $M_{kremax}$ – maximum engine torque; α – coefficient of the degree of reduction of the crankshaft rotation speed.
The negative impact of vibrations generated during the operation of a mobile chipping machine presents scientists with the challenge of creating a model that could be used to design new chipping machines with improved performance. The primary objective of a dynamic study is to determine the system's motion, i.e., to find independent, time-varying coordinates (degrees of freedom) that determine the position of all the masses in the system.
The displacement ( vibration - speed, -acceleration) of the engine support can be found

$m_d \frac{d^2 Q_{vd}}{dt^2} = \sum(M_{kr}/l_{kor\ val} - F_{tr} - F_s - F_{vd}) - G_d cos\zeta - G_o cos\vartheta,$ (12)

We integrate equations (12) and obtain

$$m_d \frac{dQ_{vd}}{dt} = \left[\sum(M_{kr}/l_{kor\ val} - F_{tr} - F_s - F_{vd}) - G_d cos\zeta - G_o cos\vartheta\right] t + c_1$$

$$m_d Q_{vd} = \left[\sum(M_{kr}/l_{kor\ val} - F_{tr} - F_s - F_{vd}) - G_d cos\zeta - G_o cos\vartheta\right] \frac{t^2}{2} + c_1 t + c_2$$

Angle of rotation of the driven link$\varphi_{max}$=335,145$^0$ and take the following values $M_{kr\ max}$=548,4 *Н* м.

The solution to equation (12) will have a different form depending on the correspondence between *h* and *w :*

a) first case: *h* < *w* (case of low resistance)

$Q_{vd} = A\, e^{-h\, t} sin(\omega_0 t + \varphi)$ (13)

$\omega_0 = \omega^2 - h^2 = 1 - \psi^2$- damping frequency of oscillations,

Where $\psi = \frac{h}{\omega}$- relative attenuation coefficient, characterizes the rate of decrease of the amplitude.

If we have *h*=*w* and $\omega_0 = 0$, i.e., free vibrations are absent. In a modern car, body vibrations occur at $\psi = 0{,}15 - 0{,}35$15-35% of the aperiodic limit.

$T = \frac{2\pi}{\omega}$ or $T = \frac{2\pi}{\omega^2 - h^2} = \frac{T_0}{1 - \frac{h^2}{\omega}}$,

Here $T_0 = \frac{2\pi}{\omega}$- the same period but without resistance.

b) the second case: if *h*=*w* then

$Q_{vd} = e^{-h\, t} \frac{Q_{0\,vd} k_2 + h + Q_{0\,vd}}{2k_2} e^{-k_2\, t} + \frac{Q_{0\,vd} k_2 - h - Q_{0\,vd}}{2k_2}$ (14)

An example of calculating the natural oscillations of the sprung mass for a mobile machine of classical design.

$m = 250\ kg, k_a = 2{,}98\ 2\ \ 0{,}255\ = 1{,}52\ \frac{kg\ s}{s м}$

$k_a$=1,52 $\frac{kg\ s}{sm}$, h=$\frac{k_a}{2m}$=2.98 $s^{-1}$, A=4 sm.

According to equation (14) we find $Q_{vd} = 4\ 2{,}7^{-2{,}8\ \ 0{,}1} sin(7{,}98\ 0{,}1 + 90^0)$

Below are graphs by which we can determine the natural oscillations of the sprung mass of a mobile vehicle.

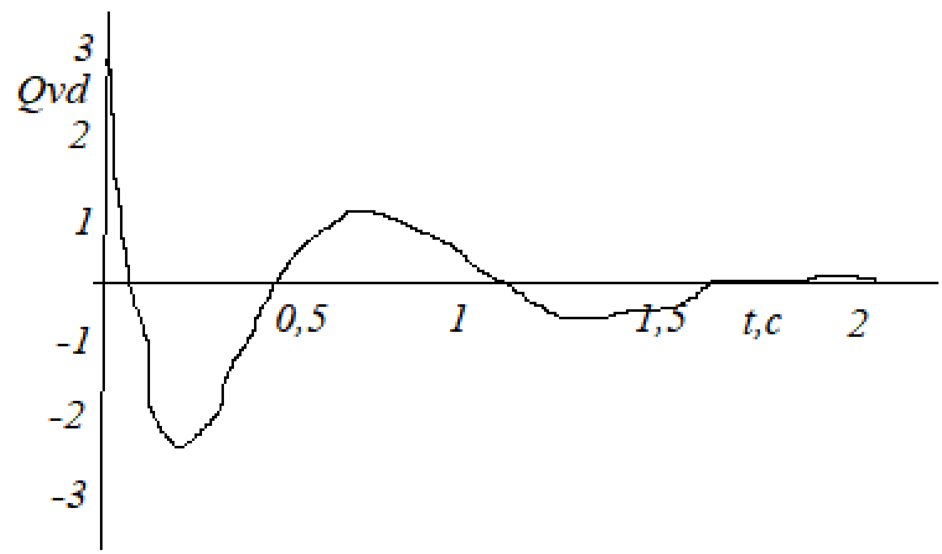


**Figure 2 - Graph of engine mount damping vibration under load from engine operation fluctuations.**

T=1.3 s-0.31 s=0.99 s.

From the graph in Figure 2 it can be seen that the oscillation period time is 1.59 sec $f = \frac{1}{T} = \frac{1}{0{,}99} = 1.01\ Gs$.

$\omega = 2\pi f = 2\ 3.14\ 1{,}01 = 6{,}34\ s^{-1}, \omega_0 = 7{,}98\ s^{-1}$,

$$\frac{7{,}98 - 6{,}34}{7{,}98} = 0{,}203\ 100 = 20{,}3\%$$

*Determining the relationship between torque and engine power*

$$G_d = \frac{g_e\ \varepsilon_N N_{nom}}{3{,}6\ 10^6} = \frac{g_e\ \varepsilon_N (N_e/(\varepsilon_M \varepsilon_\omega))}{3{,}6\ 10^6} = \frac{g_e\ \varepsilon_N \big(N_e/\big((M_e/M_{nom})\varepsilon_\omega\big)\big)}{3{,}6\ 10^6}$$

$$= \frac{g_e\ \varepsilon_N \big(N_e/\big((M_e/M_{nom})(\omega/\omega_{nom})\big)\big)}{3{,}6\ 10^6}$$

$$G_d = \frac{g_e\ \varepsilon_N \big(N_e/\big((M_{kr}/M_{nom})(\omega/\omega_{nom})\big)\big)}{3{,}6\ 10^6} = \frac{g_e\ \varepsilon_N N_e}{3{,}6\ 10^6 \big((M_{kr}/M_{nom})(\omega/\omega_{nom})\big)},$$

$$N_e = \frac{3{,}6\ 10^6 \big((M_{kr}/M_{nom})(\omega/\omega_{nom})\big) G_d}{g_e\ \varepsilon_N}.$$

**Conclusion**

An analytical relationship has been determined between the dynamics of vibration of the engine support and its torque, as well as between the torque and engine power, which will make it possible to find the displacement, vibration speed, and acceleration of the engine support depending on the torque and engine power.

The developed analytical model allows us to establish a quantitative relationship between engine torque, elastic support parameters, and the vibrational displacement of the power unit. The resulting analytical expressions can be used in the design of new engine mounting systems, the selection of optimal rubber-to-metal mount stiffness, and the calculation of damping elements.

The use of numerical methods makes it possible to predict engine vibration conditions under various operating conditions without expensive experimental studies. Practical application of the proposed method will reduce vibration levels in mobile machines, improve engine reliability, extend the service life of bearings, and reduce dynamic loads on transmission components.

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