

MODELING THE PROCESS OF INFORMATION FLOWS IN THE VERIFICATION OF TAXPAYER EVASION

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Abstract:

The most significant functions of the tax system, characterizing various aspects of its activities, include fiscal, social, regulatory, distribution, incentive and control. To implement the correct operation of each of these functions, it is necessary to exercise tax control, an important part of which is tax audits. However, earlier work has shown that performing tax audits with optimal probability is an expensive procedure. Due to the limited budget of tax authorities, such checks are not always feasible. Therefore, to increase tax collection, additional methods should be used to encourage taxpayers to pay taxes honestly. This paper examines a tax control model that considers the dissemination of information about a possible tax audit among taxpayers as an additional incentive for payments. Within the framework of the evolutionary approach to modeling, taxpayers with different risk statuses are considered as population agents: risk-prone, risk-neutral and risk-averse. The idea of information dissemination in a population is studied on the basis of a Markov process in networks with different structures. The influence of this information on the final distribution of risk statuses and tax evasion of taxpayers has been studied. A series of experiments were carried out to model networks and the process of information dissemination in them. Models with artificially introduced centers of influence and in their absence are considered. A comparative analysis of the results obtained is presented.

Keywords: tax audits, risk-propensity, information dissemination, Markov processes on the network, dynamics of opinions.

Introduction. Currently, it is impossible to underestimate the influence of information on many areas of human activity - the economy, healthcare, business processes, education, etc., including the tax system, which performs fiscal, social, regulatory, incentive and control functions. All participants in the tax system, both tax authorities and taxpayers, strive to use information about each other's behavior to their advantage. In studies [1–4], game-theoretic and probabilistic approaches to tax modeling were used, which strictly mathematically showed the ineffectiveness of a general tax audit. Based on the previously obtained results, as a tax regulation tool, this paper considers a model that includes the process of disseminating information about future tax audits among the taxable population in order to stimulate tax payments. On the one hand, tax revenues play a very important role in many social institutions and increase the efficiency of the distribution and social functions of the tax system [5], on the other hand, individual economic agents prefer to evade taxes by deliberately understating their declared income.

Let's make a few introductory assumptions. Let, according to [6], each agent chooses a behavioral strategy depending on his own risk appetite. From the point of view of the evolutionary analysis of tax evasion in society [7,8], the taxpayer's behavior is also significantly influenced by information from his social contacts, as well as parameters that determine the state of the economic environment. It is known that among the taxable population, each agent has a different risk appetite: risk aversion, risk neutrality and preference. Following earlier works devoted to the dissemination of information about tax audits [9–11], let us assume that information received from social contacts corrects the behavior of taxpayers and helps tax authorities maintain the required level of tax collection. Note that, in contrast to [9–11], this article examines different types of information and their combinations distributed among taxpayers. The population structure of taxpayers is modeled as networks with unequal topology, where the nodes are agents, and the edges reflect the social connections between them.

Previously, the problem of achieving consensus of opinions in a network was studied in 1974 by M. De Groot [12], where the model of opinion dynamics was based on the Markov process [13]. Questions about information influence based on random processes in the network were also studied in [14, 15].

In this paper, an evolutionary model of tax control is constructed and the behavior of agents is analyzed using the Markov process presented in the De Groot model. A series of experiments were conducted to illustrate possible economic scenarios.

Static model. Applying the results of [4,6], consider a population consisting of n taxpayers. We will consider an individual as a taxpayer and, accordingly, his income tax as a tax. Each agent of such a population can be characterized by the true level ξ of his income and at the same time declare it as η in the current tax period, $\eta \leq \xi$. To

simplify further reasoning, let us assume that both variables - true and declared income levels - can take three values: L, M and H, where $0 < L < M < H$ (which is a simplified modification of the models [3,16]). Because of this, the population consists of three groups: agents with a high level of true income H (number of agents in the group nH), agents with an average level M (number of the group nM) and a group of agents with a low level L (number of nL):

$$nL + nM + nH = n.$$

Thus, within the framework of the model under study, the following profiles of tax evasion are possible [3,16], formally represented as follows:

- 1) $\eta(\xi) = L(H)$;
- 2) $\eta(\xi) = L(M)$;
- 3) $\eta(\xi) = M(H)$.

Tax audits of those who declared $\eta = L$ are carried out by tax authorities with probability PL during each tax period; those who declared $\eta = M$ are checked with probability PM. There is no need to check those who declared $\eta = H$, since they are a priori not evaders. Let us assume that tax audits are absolutely effective, i.e. identify the existing deviation in all cases under consideration. If the audit reveals existing tax evasion, the evader must pay his tax arrears and a fine proportional to the level of evasion, i.e. $(\theta + \pi)(\xi - \eta)$, where the constant coefficients θ and π are the tax and penalty rates, respectively. The cost of one tax audit is also constant and amounts to c. Here are the payoff functions of taxpayers for various ways of their behavior:

$$\begin{aligned} u(L(L)) &= (1 - \theta) \cdot L, \\ u(M(M)) &= (1 - \theta) \cdot M, \\ u(H(H)) &= (1 - \theta) \cdot H, \\ u(L(H)) &= H - \theta L - PL(\theta + \pi)(H - L), \\ u(M(H)) &= H - \theta M - PM(\theta + \pi)(H - M), \\ u(L(M)) &= M - \theta L - PL(\theta + \pi)(M - L). \end{aligned}$$

In [2–4], a “threshold” value P^* of the probabilities of checking PL and PM was obtained, which, under conditions of agents being informed about their values, are critical for making a decision to evade (“Threshold Rule”). According to this rule, risk-neutral agents can change their decision to evade:

- 1) from H to M, if $PM = P^*$;
 - 2) from M to L, if $PL = P^*$;
 - 3) from H to L, if $PL = P^*$ and $PM = P^*$ simultaneously, where
- (1)

However, since the tax authorities' budget is significantly limited, achieving these audit probability values is practically impossible. Thus, tax authorities need to turn to additional means to encourage taxpayers to pay taxes in accordance with the true level

of their income. As such a method of stimulation, we will choose the dissemination of information about future audits, which overestimates the real indicators of the share of taxpayers planned for audit. Due to the model's assumption that three levels of taxpayer income are considered, two types of information can be inserted: $PL \geq P^*$ and $PM \geq P^*$.

Each agent of the system strives to increase personal well-being. At the same time, providing the population with social benefits is one of the functions of the tax system [5]. Because of this, the main quantitative characteristic of the effectiveness of taxation is the net income of the tax system, received as the total tax collected from the population, minus the total expenses aimed at ensuring tax control.

Let us determine the net income of the tax system for two different situations. In the first case, $TT R_1$ is the net income of the tax authorities in the absence of information, when the only non-evading taxpayers are risk-avoiding agents, whose share among the entire population is v_a :

(2)

In the second case, TTR_2 is the net income of the tax authorities in a situation where at the initial point in time an information dump was carried out, then the information received about the audits was disseminated by agents when communicating with each other, after which the entire system stabilized after some time (the dissemination of information was completed):

(3)

Here v_{Tinf} , V_{Tev} are, respectively, the shares of those who perceived the information and ignored it and continue to evade at the final moment (in the stationary state of the system), $V_0 inf$ is the size of information dumping (the share of those informed at the initial time $V_0 inf = V_0 inf(t_0)$), C_{inf} is the specific the price of such stuffing. At the same time, one of the main assumptions of the model is that stuffing costs the tax authorities significantly less than a tax audit: $c_{inf} \ll c$.

Network model. Note that the social connections of each taxpayer can be represented in the form of networks with different topologies [9–11]. Each population agent treats the information received depending on his personal risk appetite and the surrounding social environment.

To model the process of disseminating information about checks in the network taxpayers that occurs following an information leak, we consider an algorithm based on a Markov process similar to that described in [12]. Within the framework of this model, we will explore the possibility of injecting two types of information: $PL \geq P^*$ and $PM \geq P^*$. Consequently, we get the following options for information impact: throw in both types of information alternately, thereby stimulating taxpayers not to evade only one of the groups;

throw in both types of information at the same time, incentivizing everyone to pay taxes in accordance with their true level of income.

An algorithm based on a Markov process on a network. We will consider the taxable population as a directed network $G = (N, P)$, where N is a set of economic agents (in the current study this is a finite but large set $N = \{1, \dots, n\}$), and P is a stochastic matrix of connections : its elements depend on the presence of social connections between agents that are not strictly deterministic. In the case when there is a social connection between taxpayers i and j , the element of this matrix is strictly positive: $p_{ij} > 0$ ($i, j \in N$);

Moreover, the value of this parameter is close to 1 if the i -th agent has reason to assume that the j -th agent has expert knowledge about the probability of verification, and close to 0 otherwise. Matrix P can have different modifications, which determine the corresponding options for the network structure. These options can be summarized and classified.

Let us assume that at the initial moment of time each taxpayer from N has some idea fL_{0i} and fM_{0i} about threshold for all taxpayers in the population. In accordance with the “threshold rule” the values of the audit probabilities P_L and P_M respectively. Such parameters characterize the initial (natural or psychological) propensity of the i -th agent to take risks. Moreover, we will assume that he makes a decision on tax evasion, comparing fL_{0i} and fM_{0i} with the audit probability value (1), which is the sensitivity threshold for all taxpayers in the population. In accordance with the “threshold rule”, one of the formulations of which was presented in [3] and generalized in paragraph 2, we obtain that the i -th taxpayer evades paying taxes under the conditions $fL_{0i} < P^*$ and $fM_{0i} < P^*$, and if the threshold P^* is exceeded, he prefers not to take risks.

The interaction of taxpayers leads to an update of their ideas about the likelihood of an audit at each iteration:

similarly for fM_{ki} .

The interaction continues indefinitely or until the moment when, for some k , the conditions are not met for all i

(4)

and (5)

Changes at each iteration of the values of fL_{ki} and fM_{ki} lead to a revision of the risk status by the i -th agent: the smaller fL_{ki} and fM_{ki} , the higher the risk appetite of a given taxpayer.

In order to carry out information injection about the possible values of the probability of future audits, the tax authorities need to properly inform at least one agent from the population. Moreover, if there is a taxpayer in good standing on the

network whose knowledge of the audit probability value may be considered by others to be an expert opinion, tax authorities can use this to increase their influence over the information dissemination process across the network.

Let us now assume that information centers can be artificially introduced into the natural network population of economic agents (similar to “centers of influence” in [14]). Here and below, information centers mean agents seeking to “convince” others of a certain value of the probability of verification.

The role of such an information center can be performed by any of the taxpayers $l \in N$. Let S be the set of agents for which the condition $p_{lj} > 0, l \neq j$ is satisfied. Let us assign the parameter $\alpha_{\mu l}$ to the information center, expressing the degree of its confidence in the value $f_{\mu 0 l}$, where $\mu \in \{L, M\}$. Then, after implementing the information center, the updated elements of the l -th row of matrix P will have the following form:

where μ takes one of the values from the binary set $\{L, M\}$ depending on what type of information is distributed by this information center.

Using the described model, one can imagine the desire of tax authorities to inflate the perceptions of the taxpayer population about the values of the probabilities of a future audit. In this case, the parameters of the information center are $f_{\mu 0 l} = 1, \alpha_{\mu l} \approx 1$.

Numerical modeling. The proposed approach to modeling tax control, taking into account the dissemination of incentive information among taxpayers, provides many different scenarios for analysis. For this purpose, within the current framework, numerical experiments were carried out using specially prepared software product, a detailed description of the operation of which is presented in [17].

Cases of information dissemination over a network without information centers, as well as with one and two information centers distributing various types of information about the probability values of PL and PM checks were considered.

Experimental parameters. The numerical experiment is carried out under the assumption that all taxpayers have different risk appetites. Based on the results of a psychological study [15] devoted to the attitude towards risk of various social groups, we will use the following distribution: the share of risk-prone agents among the entire population of taxpayers reaches 18%, the share of risk-neutral agents is 65%, and risk-avoidants are 17%.

Statistical data on the distribution of income among the population of the Republic of Uzbekistan for 2020, taken from the official website of the State Statistics Service [16], were used as the initial parameter values. In accordance with the model under consideration, we will combine groups of the taxable population, presented by

official statistics, into three groups by income level: low, medium and high (L, M and H). Next, for each of these groups, the average income levels L, M and H are calculated as the mathematical expectations of uniform distribution and Pareto distribution and the shares of the total population corresponding to these levels (Table 1).

Table 1. Three Modeled Groups and Average Income

Group	Interval, sum. per month	Average income	Population share, %
L	Below 17000	L=8500	23.1
M	17000 - 50000	M=33500	56.0
H	Over 50000	H=100000	20.9

For all experiments carried out, according to the economic meaning inherent in the model, the following parameters were recorded:

- the value of the tax rate $\theta = 12\%$ is equal to the personal income tax rate in the Republic of Uzbekistan;
- the value of the penalty rate $\pi = 12\%$ is equal to the tax rate (thus, if evasion is detected, the tax arrears double);
- with parameters $\theta = 12\%$ and $\pi = 12\%$, the threshold value of the audit probability, calculated according to formula (1), is equal to $P^* = 0.5$, which is difficult to achieve from the position of the tax authorities' budget;
- the actual values of the probabilities of checking those who declared L and M are assumed not to exceed $PL = 0.1$ and $PM = 0.1$, respectively;
- the specific cost of one inspection $c = 747.3$ thousand soums was assumed to be equal to the minimum wage in Uzbekistan in 2020.
- the specific cost of information dumping does not exceed 10% of the specific cost of verification $c_{inf} = 10\% c = 74.73$ thousand soums within the framework of the assumption that the dissemination of information is significantly cheaper than carrying out a tax audit.
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As conditions for stopping the iterative process (4) and (5), the experiment considered the fulfillment of the inequality.

Visualization of experiments: notation. A numerical experiment based on the constructed model is associated with the modeling of a random network and a random process describing the dissemination of information. To illustrate the above Markov process, networks of size 25–30 nodes are studied, but the created software allows

experiments to be carried out for networks larger dimension. This number of nodes was chosen for ease of graphical representation. The following network shape options were considered: two-dimensional lattice, ring, random graph, and also random graph with almost isolated players. For the best visual display of all the information embedded in these networks, the following notations are used:

- 1) by true level of income - display is carried out using the network node form:
 - low income all around,
 - middle - quadrangle,
 - high - triangle;
- 2) by risk propensity - display is carried out using the color of the node:
 - risk-prone - light gray,
 - risk-neutral - black,
 - risk-avoiders - gray;
- 3) by level of tax evasion - display is carried out using a symbolic designation:
 - for average income level M:
 - non-deviating ($\eta(\xi) = M(M)$) - by squares, ,
 - evaders ($\eta(\xi) = L(M)$) - diamonds,
 - for high income level H:
 - non-deviating ($\eta(\xi) = H(H)$) - a triangle with a vertex pointing upward,
 - deviating by one step ($\eta(\xi) = M(H)$) - a triangle with a vertex directed to the left,
 - deviating by two steps ($\eta(\xi) = L(H)$) - a triangle with a vertex pointing downwards,

Experimental results. To analyze possible scenarios of behavior of taxpayers in response to information disseminated in their population, a number of renewable experiments were carried out to simulate networks of various configurations and the injection of information of various types. In order to take into account the randomness factor, for any network whose structure was represented by a random graph, a series of experiments were carried out. With each new start of the experiment under the same specified initial conditions, a new network in which the process of information dissemination was simulated. This approach allows us to evaluate the repeatability of agent behavior in a network with a specific structure.

When summarizing the results of the described modeling, a number of general trends were identified. Let's illustrate them with a few examples.

In Experiment 1, we study a population represented as a random network consisting of $n = 25$ nodes, of which 6- are low-income taxpayers, 14- are middle-income, and 5- are high-income. At the initial point in time, all but 9 risk-averse and low-income taxpayers avoid the maximum possible number of levels.

As an information dump, let's consider one agent who does not have a high influence in the network. When applying the information dissemination algorithm,

such a system reached a stable state in 7 iterations, and the result was stable when the algorithm is repeated.

Through the exchange of information, all taxpayers in a given network decide not to evade taxes. Thus, the implementation of the principle of tax fairness has been achieved [5]. But in view of the formulation of the task, it is important for us, first of all, to implement the fiscal function of the tax system. Analyzing its effectiveness, we see that in the absence of information stuffing, the net income of the tax authorities (2) is $TTR1 = 15371.10$ sum, while upon completion of the process of information dissemination, this indicator, calculated by formula (3), is equal to $TTR2 = 70645.20$ sum.

The obtained result demonstrates that the dissemination of information about tax audits increases net tax income by more than 4.5 times. This process also leads to the evolution of risk statuses of the population: at the initial point in time, the proportion of agents preferring risk was observed in the network; By the time the system stabilized, risk-averse agents retained their status, and risk-preferring agents became risk-neutral.

Thus, analyzing this example, we can come to the conclusion that the dissemination of information about future audits helps in the implementation of the incentive and fiscal functions of the tax system.

Conclusion. The constructed model for the dissemination of information about future tax audits among the taxable population and the numerical modeling carried out on its basis demonstrate that for fixed values of the model parameters, the experimental results obtained depend on the following factors. The final distribution of risk statuses of taxpayers and their tax evasions depends significantly, firstly, on the topology of the network, simulating the population of such agents, secondly, on the presence of risk statuses and tax evasions of information centers in the network under consideration, thirdly, on the types disseminated information.

The most important result of numerical modeling is that the dissemination of information about tax audits changes the final attitude towards risk of the population of economic agents, increases the level of declared taxpayer income and increases taxes collected from the population. It has also been mathematically shown that reducing the number of risk-prone agents allows achieve the implementation of one of the fundamental principles of taxation - the principle of fairness [5].

Thus, using the model constructed in the work and the scenario analysis of a random process on the network carried out on its basis, it was revealed that the dissemination of information about future tax audits among taxpayers is one of the tools to improve the quality of government management of tax processes. In the future,

the proposed approach can be adapted to existing tax systems, taking into account the peculiarities of legislation and economic practice.

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