

ASSESSMENT OF THE EFFECT OF THE STRUCTURAL SOLUTION OF A PRECAST-MONOLITHIC FRAME ON ITS LOAD-BEARING CAPACITY AND DEFORMABILITY USING A FULL FACTORIAL EXPERIMENTAL DESIGN

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Abstract:

The paper presents the procedure and results of processing the tests of precast-monolithic frame building models by the method of mathematical design of experiments. Two qualitative factors that define the structural solution – the type of frame elements (conventional monolithic elements, or a composite column in permanent shell formwork with a U-shaped precast-monolithic beam) and the type of floor slab (monolithic or precast hollow-core) – were included in a two-level full factorial (2^2) design. Tests of three 1:4 scale models formed three points of the design. Linear regression equations were derived for the first cracking load, the load at the limit deflection, the failure load and the deflection, and the significance of the coefficients was checked by Student's t-test ($p = 0.05$). For all four responses the coefficient of the frame-element type was found to be insignificant, and the only significant factor was the floor-slab type: the transition to a precast hollow-core slab reduces the failure load by 10.86 kN and increases the deflection by 2.39 mm. The indicators of the untested fourth variant were predicted by the additive model. The results provide a statistical basis for using the composite column in permanent formwork and the U-shaped beam instead of conventional monolithic elements.

Keywords: design of experiments; full factorial experiment; regression equation; Student's t-test; precast-monolithic frame; permanent formwork; floor-slab type; failure load; deflection.

Introduction

The main difficulty in the experimental justification of new structural solutions for reinforced concrete

frame buildings is the high cost and labour intensity of the tests: testing a whole frame or a large fragment of it requires months of specimen preparation and a special test rig [1], [2]. For this reason, the number of test points in such studies is very limited, and whether the observed differences are random or systematic cannot be established by a simple percentage comparison.

Methods of mathematical design of experiments [3]–[5] make it possible to isolate the effect of each factor from a small number of tests and to assess its statistical significance. Although these methods are widely used in building materials technology, they are rarely applied to tests of whole frame models, especially when the factors are qualitative (the type of structural solution).

The author's previous works [6]–[8] describe the modelling conditions, the test programme and the results of tests of individual elements of precast-monolithic frame models with columns in permanent reinforced concrete shell formwork. The aim of this paper is to process the results of the frame model tests as a full factorial experimental design and to give a quantitative and statistical assessment of the effect of the frame-element type and the floor-slab type on the load-bearing capacity and deformability.

Materials and Methods

Test object

Three building models of identical geometry were tested (Fig. 1): M-1 – a conventional monolithic frame; M-2 – a composite column in permanent reinforced concrete shell formwork, a U-shaped precast-monolithic beam and a monolithic floor slab; M-3 – the same column and beam, but with a precast hollow-core floor slab. The models were made at a scale of 1:4, with two spans (2×1300 mm), two storeys (storey height 800 mm) and a column and beam cross-section of 100×100 mm. All models were made from a single batch of concrete (actual class B30) with the same reinforcement (A400) and tested under the same loading programme [6], [9].

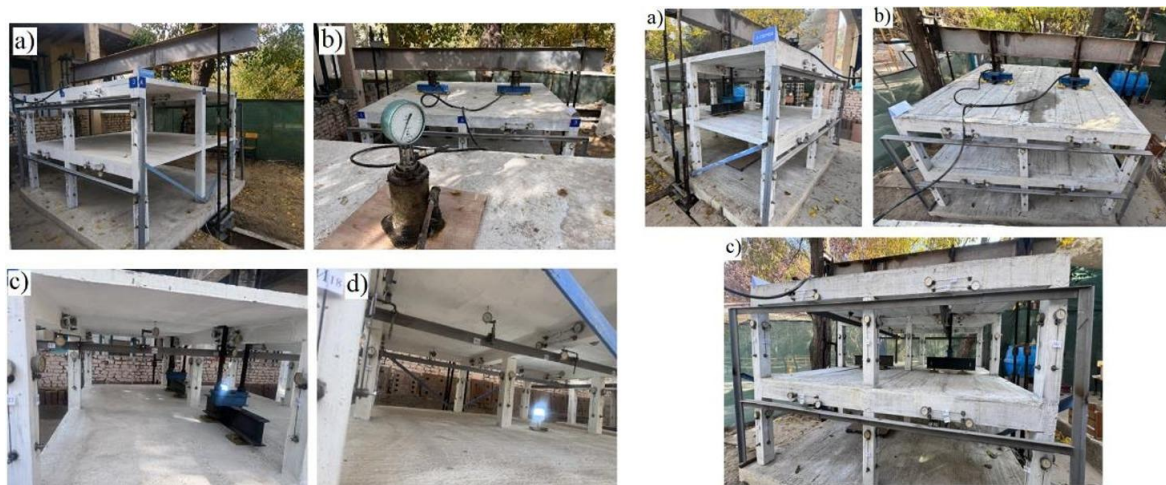


Figure 1. Testing: left – model M-1 (conventional monolithic frame); right – model M-3 (frame with a precast hollow-core floor slab)

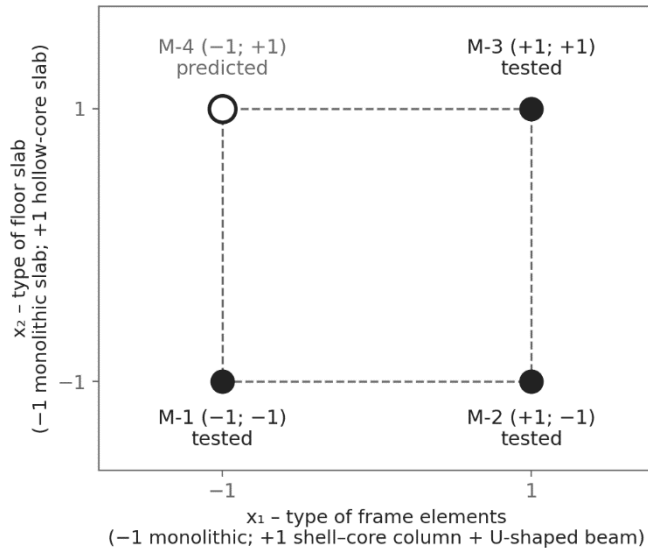
Factors and design matrix

Three structural features affect the behaviour of the frame: the column type, the beam type and the floor-slab type. The column in permanent shell formwork and the U-shaped precast-monolithic beam form a single structural system and are not used separately in practice; therefore, they were combined into one generalised factor. The result is a two-factor, two-level (2^2) design (Table 1). Since the factors are qualitative, their levels were denoted by coded values (-1 and $+1$) [3], [4].

Table 1. Factors and their levels

Factor	Symbol	Lower level (-1)	Upper level (+1)
Type of frame elements (column and beam)	x_1	Conventional monolithic column and beam	Composite column in permanent shell formwork and U-shaped precast-monolithic beam
Type of floor slab	x_2	Monolithic slab (40 mm)	Precast hollow-core slab (55 mm)

The full 2^2 design comprises four combinations, three of which were implemented experimentally (Fig. 2): M-1 (-1; -1), M-2 (+1; -1) and M-3 (+1; +1). The fourth combination (-1; +1) – a conventional monolithic frame with a precast hollow-core floor slab – was not tested and was later predicted by the regression model.

Figure 2. Layout of the 2^2 design in coded coordinates: ● – tested models, ○ – predicted variant

Four indicators were taken as responses (y): the first cracking load P_{cr} , the load at the limit deflection P_f , the failure load P_u , and the deflection f in the first span of the second storey of the middle frame (axis B) at the load level $P = 101.54 \text{ kN}$ ($\approx 0.75 \cdot P_u$), measured by dial gauge D-5 (Fig. 3). Here P is the force in one jack, and the total load applied to the model equals $2 \cdot P$. The load corresponding to the limit crack width was not used as a response, since it was recorded only in model M-3. The design matrix is given in Table 2.

Table 2. Design matrix and test results

No.	Model	x_0	x_1	x_2	P_{cr} , kN	P_f , kN	P_u , kN	f , mm
1	M-1	+1	-1	-1	43.62	122.15	130.50	3.14
2	M-2	+1	+1	-1	43.62	125.84	134.12	3.39
3	M-3	+1	+1	+1	36.38	104.42	123.26	5.78

No. 5 – Deflection Indicator Measurements



Figure 3. Comparison of vertical deflections measured by dial gauge D-5 (series 1 – M-1, series 2 – M-2, series 3 – M-3)

Regression model and significance criterion

The response was sought in the form of a linear regression equation without interaction:

$$y = b_0 + b_1 \cdot x_1 + b_2 \cdot x_2,$$

where b_0 is the intercept, and b_1 and b_2 are the linear effects of the factors. Since there are three tests and three unknown coefficients, the system has an exact solution:

$$b_1 = (y_2 - y_1)/2; \quad b_2 = (y_3 - y_2)/2; \quad b_0 = (y_1 + y_3)/2.$$

Such a design belongs to the class of saturated designs: the regression equation passes exactly through the test points, and the scatter cannot be estimated from the model residuals. Therefore, the reproducibility error was determined from auxiliary specimens tested in parallel [4], [5]. Two series of column specimens were tested, three in each: series 1 – 310, 320 and 325 kN ($S = 7.64$ kN); series 2 – 330, 340 and 348 kN ($S = 9.02$ kN) [7]. From the pooled variance (degrees of freedom $f = 4$), $S\{y\} = 8.36$ kN and the coefficient of variation $C_v = 2.54$ %. For each response, the reproducibility error and the coefficient error were determined as:

$$S\{y\} = C_v \cdot \bar{y}; \quad S\{b_i\} = S\{y\}/\sqrt{N},$$

where $\bar{y}$ is the mean value of the response and $N = 3$ is the number of tests. A coefficient was considered significant if $|b_i| > t_p(f) \cdot S\{b_i\}$; for $p = 0.05$ and $f = 4$, $t_p = 2.776$ [10].

Results

The regression coefficients and the results of the significance check are given in Table 3. The ratio of each coefficient to its significance threshold is shown in Fig. 4.

Table 3. Regression coefficients and their significance

Response	b_0	b_1	b_2	$S\{y\}$	$t_p \cdot S\{b_i\}$	Conclusion
P_{crc} , kN	40.00	0.00	-3.62	1.05	1.68	only b_2 significant
P_f , kN	113.29	+1.85	-10.71	2.98	4.79	only b_2 significant
P_u , kN	126.88	+1.81	-5.43	3.28	5.27	only b_2 significant
f (D-5), mm	4.46	+0.13	+1.20	0.10	0.17	only b_2 significant

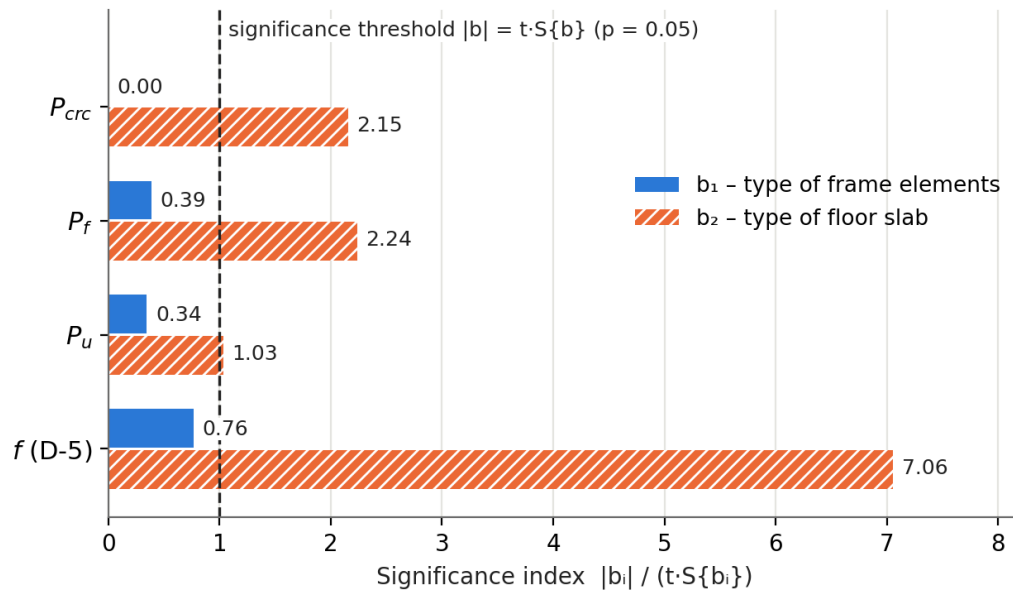


Figure 4. Significance index of the regression coefficients $|b_i|/(t_p \cdot S\{b_i\})$; values to the right of the line are significant

The following follows from the results obtained.

Type of frame elements (x_1). For all four responses the coefficient b_1 is below the significance threshold: the index is 0.00 (P_{crc}), 0.39 (P_f), 0.34 (P_u) and 0.76 (f). Hence, the differences observed when the conventional monolithic column and beam are replaced by a composite column in permanent shell formwork and a U-shaped precast-monolithic beam (+3.62 kN, i.e. +2.8 %, in the failure load) lie within the experimental scatter. For the first cracking load $b_1 = 0$: in both solutions the first crack appeared at exactly the same load step.

Type of floor slab (x_2). For all four responses the coefficient b_2 is significant. The transition from a monolithic slab to a precast hollow-core slab (a change in x_2 of 2 units) has the following effect: the first cracking load decreases by $2 \cdot |b_2| = 7.24$ kN, the load at the limit deflection by 21.42 kN and the failure load by 10.86 kN, while the deflection increases by 2.39 mm. The strongest effect is observed for the deflection (index 7.06), i.e. the floor-slab type mainly governs the stiffness of the frame.

For the failure load, the index of the coefficient b_2 is 1.03, only slightly above the significance threshold. This shows that the effect of the floor-slab type on the failure load exists, but is much weaker than its effect on the deflection and cracking.

Prediction of the untested variant

For the fourth combination ($x_1 = -1$; $x_2 = +1$), the indicators were calculated by the additive model using the expression $y_4 = b_0 - b_1 + b_2 = y_1 + y_3 - y_2$ (Table 4).

Table 4. Experimental and predicted indicators

Indicator	M-1 (test)	M-2 (test)	M-3 (test)	M-4 (prediction)
$P_{cr}, \text{ kN}$	43.62	43.62	36.38	36.38
$P_t, \text{ kN}$	122.15	125.84	104.42	100.73
$P_u, \text{ kN}$	130.50	134.12	123.26	119.64
$f (D-5), \text{ mm}$	3.14	3.39	5.78	5.53

According to the prediction, when a precast hollow-core floor slab is used in a conventional monolithic frame (M-4), the failure load is expected to be 3.62 kN lower than that of M-3. In other words, even in combination with a precast slab, the composite column and the U-shaped beam perform no worse than conventional monolithic elements.

Discussion

The statistical assessment complements the simple percentage comparison of the frame model tests in an important respect. Taken by itself, the failure load of model M-2, which is 2.8 % higher than that of the reference model, could lead to the conclusion that “the new solution is stronger”. The regression analysis, however, shows that this difference lies within the random scatter. Therefore, the correct conclusion is not that “the new solution is stronger” but that “the new solution is equal in strength to the conventional one”. From the design point of view this conclusion is even more important, since it justifies calculating the precast-monolithic frame by the same code method as the conventional monolithic frame [11], [12].

The significant effect of the floor-slab type agrees with the known conclusions on the importance of the joint between the precast and cast-in-situ parts of precast-monolithic structures [1], [13], [14]: the connection of precast hollow-core slabs to the beam is less stiff than that of a monolithic slab, which manifests itself primarily in the deflection.

Three limitations should be taken into account when interpreting the results. First, the design is saturated: the coefficients were determined from three tests, and no degrees of freedom remain for checking the adequacy of the model by Fisher’s criterion. Second, the reproducibility error was obtained not from whole frame models, but from individual column specimens made by the same technology; the scatter at the frame level may be larger, which strengthens the conclusion that b_1 is insignificant but weakens the borderline significance of b_2 for P_u . Third, the additive model does not account for the interaction of the factors; testing variant M-4 would make it possible to verify this assumption and to complete the full 2^2 design [15].

Conclusions

The tests of the frame models were formalised as a two-factor, two-level (2^2) full factorial experimental design; the three tested models formed three points of the design, and the regression coefficients were determined from the saturated design.

For the first cracking load, the load at the limit deflection, the failure load and the deflection, the coefficient of the frame-element type (b_1) is insignificant by Student’s t-test. Hence, the composite column in permanent shell formwork and the U-shaped precast-monolithic beam perform statistically

on a par with the conventional monolithic column and beam.

The only significant factor is the floor-slab type: the transition to a precast hollow-core slab reduces the failure load by 10.86 kN and increases the deflection by 2.39 mm; the strongest effect is on stiffness.

For the untested variant “conventional monolithic frame + precast hollow-core slab”, the additive model predicts a failure load of 119.64 kN; testing this variant is recommended to complete the design and to check the adequacy of the model.

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