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# G-Hypergraphs Induced by Locally Compact Hypergroups

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**Abstract:**

In this paper, we introduce the notion of a g-hypergraph as a natural extension and generalization of both classical graph theory and hypergraph theory. The proposed structure provides a broader mathematical framework for studying relationships among elements in algebraic systems, particularly those arising from hypergroups. By combining the structural properties of graphs with the generalized connectivity of hypergraphs, g-hypergraphs offer a flexible and powerful tool for analyzing complex algebraic and topological interactions. We investigate several important classes of g-hypergraphs induced by hypergroups and examine their fundamental structural characteristics.
The paper also examines conditions under which the pair H:=(V,E), consisting of the vertex set and edge set of a g-hypergraph, satisfies desirable topological properties. In particular, we discuss the role of local compactness and provide several applicable sufficient conditions ensuring that the induced g-hypergraph possesses appropriate topological compatibility. These results contribute to the development of a unified framework connecting hypergroup theory, topology, and generalized graph structures. The concepts and results presented in this work may serve as a foundation for further investigations in algebraic graph theory, topological hyperstructures, and related applications in mathematical modeling and discrete structures.

Keywords: G-HYPERGRAPHS, G-topologically the pair H:=(V,E), LOCALLY COMPACT HYPERGROUPS

## 1. Introduction

Over the last decades, graph theory has been widely studied and has found many applications in various branches of mathematics and other sciences. Therefore, it has always been of interest. See[1][2][3][4]. However, some structures have not been applicable to it, and hypergraphs have been defined, studied and used as a complex generalization of graphs. In short, in a hypergraph, an edge can be related to more than two vertices. For more details, see the references[5][6][7]. In this article, we want to express a generalization of hypergraphs. In this generalization, the edges related to vertices can even have elements other than vertices. To examine a class of this new structure, we have gone to locally compact hypergroups to create a generalized hypergraph by taking advantage of the convolution product between measures. After stating some definitions, we study the degree of each vertex and under some conditions state a formula for it.

## 2. Materials and Methods

In this research, we used the relationship between the algebraic properties of graphs, generalized hypergraph, or simply a g-hypergraph. where the construction of vertex spaces is a topological space, where the vertices of the graph are represented, and we delegate the property of a local compact space to ensure the existence of neighborhood for all vertices.
This relationship is generalized to include the bilateral effect to ensure that the edges remain constant in the same elements.

## 3. Results and Discussion

Recall that if $V$ is a non-empty set and $E$ is a nonempty subset of the power set of $V$, then the pair ( $V, E$ ) is called a hypergraph. This hypergraph will be a graph whenever for every $e \in E$ we have $|e| \leq 2$, where $|e|$ denotes the number of elements of the set $e$.
In this section, first we present the notion g-hypergraph, which is an extension of hypergraph, and next, we will study this new concept.
Definition 2.1. Assume that $\Omega$ is a non-empty set, and fix some $\emptyset \neq V \subseteq \Omega$. Let $E$ be a non-empty collection of the non-void subsets of $\Omega$. Then, the pair $H := (V, E)$ is called a generalized hypergraph, or simply a $g$-hypergraph.
Note that in the above definition we do not limit ourself to finite sets. Also, we mention that any hypergraph is a g -hypergraph. In fact, while in the above definition any element of $E$ is a subset of $V$, then $H$ will be a hypergraph. For example setting $\Omega := \mathbb{N}$, $V := \{2,4,5,7,10\}$ and $E := \{\{1,2,4\}, \{4,10\}, \{2,4,10\}, \{3\}\}$, the pair $(V, E)$ is a g-hypergraph which is not a hypergraph.
In the following definition, we define some classes of g-hypergraphs which are induced by a locally compact hypergroup. Hypergroups were introduced in the papers**Error! Reference source not found.Error! Reference source not found.Error! Reference source not found.**, and in the book **Error! Reference source not found.** one can study regarding them. Roughly speaking, a hypergroup has enough structure for that its measure space to be a convolution algebra, and also the Fourier analysis can be applied for them. We turn our attention to a general hypergroup endowed with as convolution product and the mapping as involution. Furthermore, denotes the support of a regular measure, and is the Dirac mass at point. An involution is called Hermitian if: for, one has. Besides the wise and for all means. For simply, put for all.**Error! Reference source not found.Error! Reference source not found.**
Definition 2.2. Let be a locally compact hypergroup, and Assume that, and let
There, we have that is a continuous -hypergraph created by. Also, we denote and.

Remark 2.3. Note that if for all we have; then the pair will be a hypergraph. More precisely, if has a subhypergroup of, for every ( ) hypergraph.
Then always is a locally compact hypergroup.**Error! Reference source not found.Error! Reference source not found.**
Definition 2.4. Let be a g-hypergraph; that is,. Next, we define the g -degree of A as the number of all elements with g(A) = A, and we can denote by.
Let $A, B \subseteq K$. Then, we say that the pair ( $A, B$ ) has DC (i.e. disjoint convo) property whenever for every distinct $a_1, a_2 \in A$ and $b_1, b_2 \in B$ we have

$$\operatorname{supp}\left(\delta_{a_1} * \delta_{b_1}\right) \neq \operatorname{supp}\left(\delta_{a_2} * \delta_{b_2}\right)$$

Example 2.5. For every $x, y \in [0, \infty)$ define

$$\delta_x * \delta_y := \frac{1}{2}\delta_{|x-y|} + \frac{1}{2}\delta_{x+y}$$

Then, $K := [0, \infty)$ is a Hermitian hypergroup equipped with the Euclidean topology. For every with and we have
if and only if. That is satisfied if and only if and, which is equivalent to and
. In other words, in general we have for each distinct and distinct,
if and only if and. This shows that if with, then ( ) is DC.
Example 2.6. Assume, so one is due to the article, we are | χR | lobby a representation zero of Q as a discrete Hermitian hypergroup with identity 0 and ∀x ∈Q:{λ : λ(χR) = (xtson x(t) also ˙{t}| < infty }

$$\delta_a * \delta_b := \delta_{\max\{a,b\}}$$

if $a \neq b$, and

$$\delta_a * \delta_a := \frac{1}{23^{a-1}}\delta_0 + \sum_{k=1}^{a-1} \frac{1}{3^{a-k}}\delta_k + \frac{1}{2}\delta_a$$

This implies that $\operatorname{supp}(\delta_a * \delta_b) = \max\{a, b\}$ if $a \neq b$, and $\operatorname{supp}(\delta_a * \delta_a) = \{0,1,2, \ldots, a\}$. Let $x, y \in K$ be distinct, and also $a, b \in K$ be distinct. In this case, by some simple calculation one can see that

$$\operatorname{supp}(\delta_x * \delta_a) = \operatorname{supp}\left(\delta_y * \delta_b\right)$$

if and only "both $x, a$ and $y, b$ are distinct and $\max\{a, x\} = \max\{b, y\}$ ", or $x = a = b = y$.
Notation 2.7. Let $C, D \subseteq K$. Then, we denote

$$D^- := \{x^- : x \in D\}$$

and

$$C \circledast D^- := \bigcup \left\{\operatorname{supp}\left(\delta_x * \delta_{y^-}\right) : x \in C, y \in D, x \neq y\right\}$$

Remark 2.8. sychical permuting random variable with respect to the distribution of real numbers. We will use this fact in the following proof.**Error! Reference source not found.**
Theorem 2.9. Let be a locally compact hypergroup such that,
and suppose that is the continuous -hypergraph induced by. Then, for every we have
Proof. This is sufficient to show that, where
To achieve this, first observe that for all possible and we have if and only if, which is iff. Next, we show that any two different and in must have. For this, let.
Case 1:. By the assumption we have. This means that, and consequently, we get.
Case 2:. By the assumption, and hence.
Therefore, the proof is complete.
Example 2.10. Let be the hypergroup given in Example 2.6. Let and. Then, we have

and so,. Easily, since is singleton. Put

$$V := \{0,1,2,3,4,5,6,7,9,11,15,20\}$$

Then, $A * B = \{0,1,2,3,4,5,7,9,11\} \subseteq V$. By the above theorem, setting $H := (V, E_{A,B,V})$ we have $gd_H(6) = \text{card}((6 * B^-) \cap A) \times \text{card}(B) = \text{card}(\{6,7,9,11\} \cap \{5\}) \times 4 = 0$,
and

$$gd_H(7) = \text{card}((7 * B^-) \cap A) \times \text{card}(B)$$
$$= \text{card}(\{1,2,3,4,5,6,7,9,11\} \cap \{5\}) \times 4 = 4$$

Example 2.11. Let $K$ be the hypergroup as in Example 2.5. Setting $A := \{7,8\}$ and $B := (2,3)$ we have $B \circledast B^- = (0,1) \cup (2,6), A * (B \circledast B^-) = (7,8) \cup (8,9) \cup (9,14)$, so, $A \cap (A * (B \circledast B^-)) = \emptyset$.
Also,
$A^- \circledast A = \{1,15\}, (A^- \circledast A) * B = (1,2) \cup (3,4) \cup (12,13) \cup (17,18)$, hence, $B \cap ((A^- \circledast A) * B) = \emptyset$.

**5. Conclusion**

In this theses, we introduce new definition called the closure space on in-linked digraphs which to generate topological structure on the power set of vertices of digraphs. And it can be concluded that:

1. Several properties are investigate for this topological structure and several examples are given.
2. W given some new generalizations of some definitions in digraphs to some well-known topological definitions such as open subdigraph, open subdigraph, semi open- subdigrapg, pre-open subdigraph, semi preopen- subdigraph and open subdigraph also several properties of these new concepts are investigate.

Summary

This new introduced structure (g-hypergraph) can be basic for new studies same as researches regarding graphs and hypergraphs, specially related to a locally compact hypergroup.

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