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Multiplication, Division and Exponentiation of Complex Numbers Given in Trigonometric Form

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Abstract:

Several forms of complex analysis are shown in this article, including indexed, trigonometric, forms and theorems are proved.

Keywords: *Complex numbers, Muovur formula, Trigonometric forma.*

It is convenient to work with their algebraic expression rather than performing addition and subtraction operations on complex numbers. The trigonometric expression of complex numbers is useful for multiplying, dividing and raising complex numbers.

Theorem 1. To multiply two complex numbers in trigonometric form, their modules are multiplied, and their arguments are added, i.e.

$$z_1 \cdot z_2 = r_1(\cos\varphi_1 + i\sin\varphi_1)\cdot r_2(\cos\varphi_2 + i\sin\varphi_2) = r_1 \cdot r_2[\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2)]$$

proof. $z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1),\ z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2)$

$$z_1 \cdot z_2 = r_1(\cos\varphi_1 + i\sin\varphi_1)\cdot r_2(\cos\varphi_2 + i\sin\varphi_2) =$$
$$= r_1 \cdot r_2(\cos\varphi_1 \cdot \cos\varphi_2 + i\cos\varphi_1 \cdot \sin\varphi_2 + i\sin\varphi_1 \cdot \cos\varphi_2 - \sin\varphi_1 \cdot \sin\varphi_2) =$$
$$= r_1 \cdot r_2[(\cos\varphi_1 \cdot \cos\varphi_2 - \sin\varphi_1 \cdot \sin\varphi_2) + i(\sin\varphi_1 \cdot \cos\varphi_2 + \cos\varphi_1 \cdot \sin\varphi_2)] =$$
$$= r_1 \cdot r_2[\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2)].$$

1). $$2(\cos 40^0 + i\sin 40^0)\cdot(\cos 20^0 + i\sin 20^0) = 6(\cos(40^0 + 20^0) + i\sin(40^0 + 20^0)) =$$
$$= 6(\cos 60^0 + i\sin 60^0) = 3(1 + i\sqrt{3}).$$

$$2).\ \frac{2}{\sqrt{3}}\left(\cos 79^0 + i\sin 79^0\right)\cdot\frac{\sqrt{2}}{2}\left(\cos 101^0 + i\sin 101^0\right) = \frac{\sqrt{2}}{3}\left(\cos 1180^0 + i\sin 180^0\right) =$$
$$= \frac{\sqrt{2}}{3}\left(-1 + 0\cdot i\right) = -\frac{\sqrt{2}}{3}.$$

1-result. The modulus of the product of a complex number *n* is equal to the product of the modules of the given complex number *n*, and the argument is a complex number equal to the sum of its arguments, i.e.

$$r_1\left(\cos\varphi_1 + i\sin\varphi_1\right)\cdot r_2\left(\cos\varphi_2 + i\sin\varphi_2\right)\cdot\ldots\cdot r_n\left(\cos\varphi_n + i\sin\varphi_n\right) =$$
$$= r_1\cdot r_2\cdot\ldots\cdot r_n\left[\cos\left(\varphi_1 + \varphi_2\cdot\ldots\cdot\varphi_n\right) + i\sin\left(\varphi_1 + \varphi_2\cdot\ldots\cdot\varphi_n\right)\right].$$

Theorem 2. The ratio of one complex number to another non-zero complex number is the complex number whose modulus is equal to the ratio of the modules of the divisor and divisor complex numbers, and the argument is the difference of the arguments of these numbers, i.e.

$$\frac{z_1}{z_2} = \frac{r_1\left(\cos\varphi_1 + i\sin\varphi_1\right)}{r_2\left(\cos\varphi_2 + i\sin\varphi_2\right)} = \frac{r_1}{r_2}\left[\cos\left(\varphi_1 - \varphi_2\right) + i\sin\left(\varphi_1 - \varphi_2\right)\right].$$

proof. $z_1 = r_1\left(\cos\varphi_1 + i\sin\varphi_1\right),\ z_2 = r_2\left(\cos\varphi_2 + i\sin\varphi_2\right),\ z_2 \neq 0$

$$\frac{z_1}{z_2} = \frac{r_1\left(\cos\varphi_1 + i\sin\varphi_1\right)}{r_2\left(\cos\varphi_2 + i\sin\varphi_2\right)} = \frac{r_1}{r_2}\frac{\left(\cos\varphi_1 + i\sin\varphi_1\right)\left(\cos\varphi_2 - i\sin\varphi_2\right)}{\left(\cos\varphi_2 + i\sin\varphi_2\right)\left(\cos\varphi_2 - i\sin\varphi_2\right)} =$$
$$= \frac{r_1}{r_2}\frac{\left(\cos\varphi_1 + i\sin\varphi_1\right)\left(\cos\left(-\varphi_2\right) - i\sin\left(-\varphi_2\right)\right)}{\cos^2\varphi_2 - i^2\sin^2\varphi_2} =$$
$$= \frac{r_1}{r_2}\left\{\left(\cos\varphi_1\cdot\cos\left(-\varphi_2\right) - i\sin\varphi_1\sin\left(-\varphi_2\right)\right) + i\left(\cos\varphi_1\cdot\sin\left(-\varphi_2\right) + \cos\left(-\varphi_1\right)\sin\varphi_1\right)\right\}$$

Examples.

1)
$$\frac{2\left(\cos 130^0 + i\sin 130^0\right)}{3\left(\cos 70^0 + i\sin 70^0\right)} = \frac{2}{3}\left[\cos\left(130^0 - 70^0\right) + i\sin\left(130^0 - 70^0\right)\right] =$$
$$= \frac{2}{3}\left(\cos 60^0 + i\sin 60^0\right) = \frac{1}{3}\left(1 + i\sqrt{3}\right).$$

$$\frac{5\left(\cos 18^0 + i\sin 18^0\right)}{9\left(\cos 48^0 + i\sin 48^0\right)} = \frac{5}{9}\left[\cos\left(-30^0\right) + i\sin\left(-30^0\right)\right] =$$
$$= \frac{5}{9}\left(\cos 30^0 - i\sin 30^0\right) = \frac{5}{9}\left[\frac{\sqrt{3}}{2} - \frac{i}{2}\right] = \frac{5}{18}\cdot\left(\sqrt{3} - i\right).$$

Theorem 3. To raise a complex number to *n* - degree, it is enough to raise its modulus to this degree, and to multiply its argument by the exponent of the degree, i.e.

$$\left[r\left(\cos\varphi + i\sin\varphi\right)\right]^n = r^n\left(\cos n\varphi + i\sin n\varphi\right).$$

This equality is called Muavr's formula. By opening the parentheses on the left side according to Newton's construction, and then equating the real and abstract parts of both sides of the equations, various trigonometric formulas can be derived.

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